External assessment trial — Mathematics B

Examiner's report

August 2017

Queensland Government | Queensland Curriculum & Assessment Authority

160689

For all Queensland schools

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Contents

Introduction

In Semester 1, 2017 the Queensland Curriculum and Assessment Authority (QCAA) trialled external assessment in Year 11 Mathematics B.

The *External assessment trial — Semester 1, 2017* familiarised schools and students with subject-based external assessments and tested processes for their delivery. The trialled assessment was aligned to the *Mathematics B Senior Syllabus 2008* (amended 2014) and developed in consultation with subject experts from schools and universities. It was administered under secure conditions and marked externally.

A total of 5928 students from 118 participating schools were involved in the Mathematics B external assessment trial, and 90 teachers participated in the online marking operation.

This report provides information on the Mathematics B external assessment trial specifications, students' performance characteristics and aggregated results from all participating schools. The assessment was formative and provided an alternative to a task already being undertaken at participating schools.

The QCAA appreciates schools' participation in the external assessment trial. The teachers and students who participated in the trial have made a valuable and significant contribution to Queensland's new system of senior assessment.

To provide feedback on the trial or further advice, please contact Assessment Operations on telephone 1300 381 575 or by email at seaops@qcaa.qld.edu.au.

Claude Jones

Director, Assessment and Reporting Division
Queensland Curriculum and Assessment Authority

Overall commentary

The Mathematics B external assessment trial was a QCAA-developed examination conducted under supervised conditions on 1 June 2017.

The assessment was devised from the *Mathematics B Senior Syllabus 2008* (amended 2014). It required students to demonstrate their understanding of Introduction to functions, Periodic functions and applications, and Applied statistical analysis. Students were assessed on three criteria:

- *Knowledge and procedures (KAP)*
- *Modelling and problem solving (MAP)*
- *Communication and justification (CAJ).*

Schools were provided with supplementary materials to support the development of teaching and learning experiences.

The assessment contained two papers. Paper 1 was technology free and Paper 2 was technology active. There were 63 items, grouped into 4 sections — Section A, B, C and D. Sections A and B formed Paper 1, while Sections C and D formed Paper 2. Section A consisted of 5 questions requiring a single-word (or single-term) response and assessed *KAP*. Section B consisted of eight short-response questions assessing *KAP, MAP* and *CAJ*. Section C consisted of 10 multiple-choice questions, assessing *KAP* and *MAP*. Section D consisted of six short-response questions assessing *KAP, MAP* and *CAJ*.

In *KAP*, 68% of students achieved a passing grade — 19% at A standard, 29% at B standard, and 20% at C standard. In *MAP*, 74% of students achieved a passing grade — 17% at A standard, 17% at B standard, and 40% at C standard. In *CAJ*, 76% of students achieved a passing grade — 22% at A standard, 27% at B standard, and 27% at C standard.

Statistics in this report may have been rounded, resulting in totals not equal to 100%.

Figure 1: Statewide student results — *Knowledge and procedures*

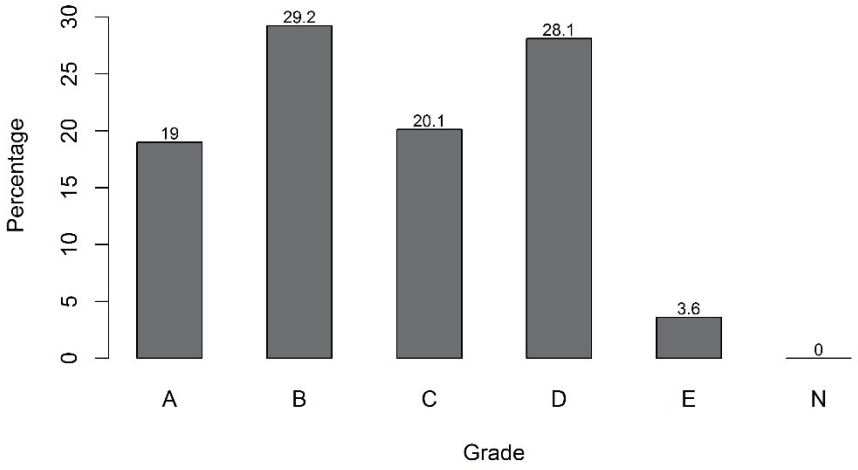


Figure 2: Statewide student results by gender — *Knowledge and procedures*

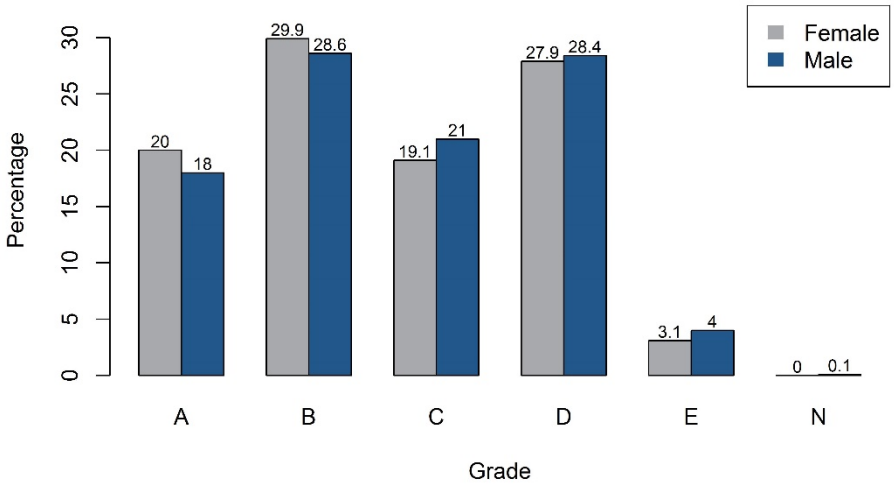


Figure 3: Statewide student results — *Modelling and problem solving*

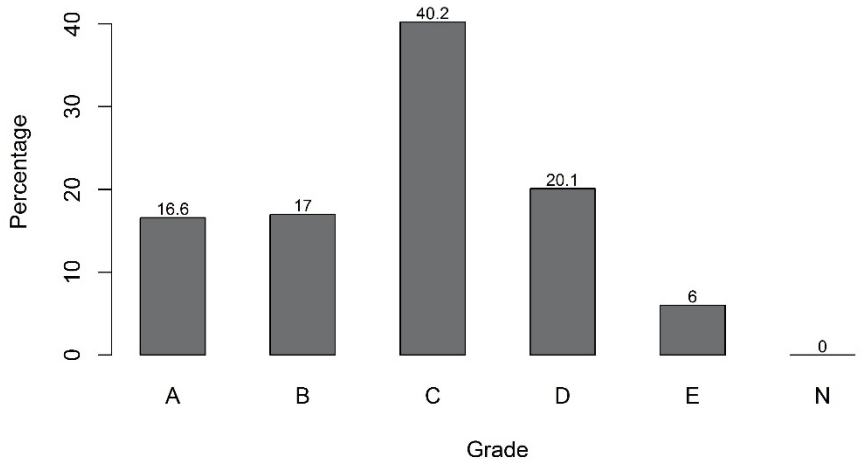


Figure 4: Statewide student results by gender — *Modelling and problem solving*

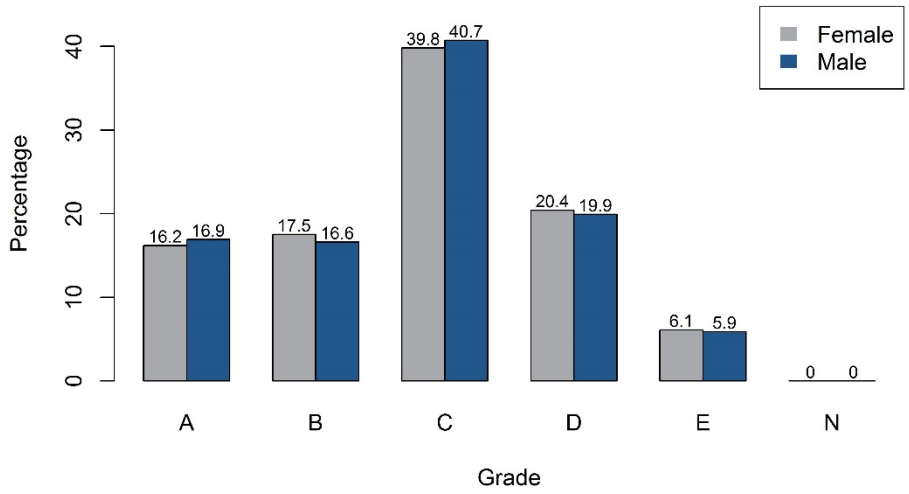


Figure 5: Statewide student results — *Communication and justification*

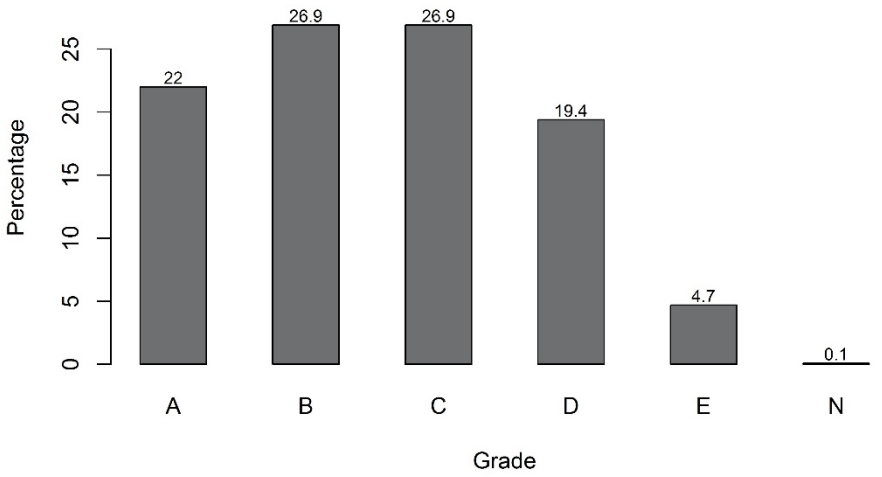
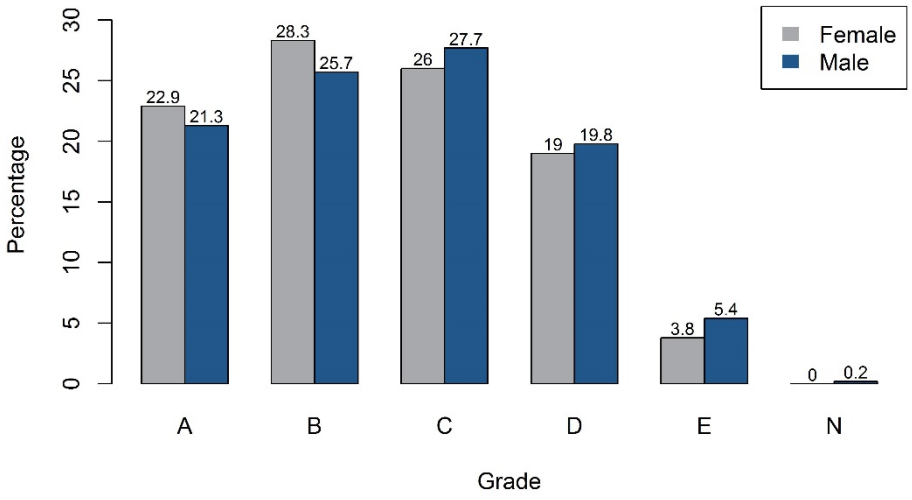


Figure 6: Statewide student results by gender — *Communication and justification*



Sample responses and commentaries

The following section provides commentary on sample responses to the questions in the external assessment trial. The samples provide indicative student responses to each question. These responses have not been corrected for accuracy and are not necessarily exemplary.

Section A

Question 1

This question was a simple routine *Knowledge and procedures* question (1 mark).

For a certain angle θ, where $0° \leq \theta < 360°$, $\sin\theta$ is negative and $\cos\theta$ is positive.

Which quadrant does angle θ belong to?

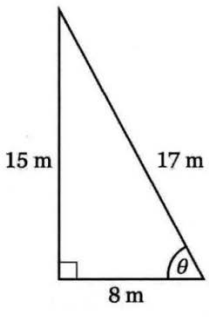
angle θ = Quadrant 4 *[1 mark]*

This question required students to determine which quadrant an angle must be located in, to satisfy given sign conditions placed on the trigonometric functions sine and cosine. A single-word (or single-term) response such as 'four' or 'quadrant 4' was expected.

59% of students achieved success with identifying the fourth quadrant.

Question 2

This question was a simple routine *Knowledge and procedures* question (1 mark).



Not to scale

Using the triangle shown, with respect to the angle labelled θ, which trigonometric ratio does the ratio $\frac{8}{17}$ represent?

$\frac{8}{17}$ represents cos θ, or $\frac{A}{H}$ *[1 mark]*

Students were required to select the appropriate definition of a trigonometric ratio in the context of a labelled right-angled triangle. A single-word response such as 'cosine' was expected.

Almost 90% of students correctly identified the cosine ratio.

Question 3

This question was a simple routine *Knowledge and procedures* question (1 mark).

What statistical name is given to the spread of data found in the box section of any boxplot (box-and-whisker plot)?

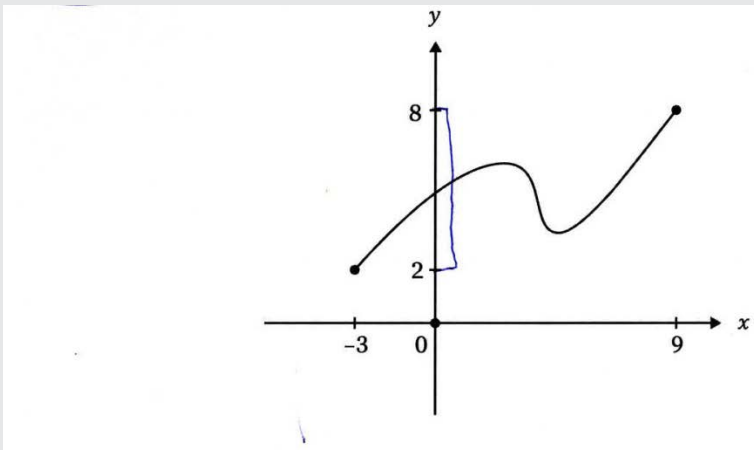
interquartile range [1 mark]

Students were required to recall the statistical definition that is described by the box section of any boxplot. A single-word (or single-term) response such as 'interquartile range' was expected.

56% of students achieved success with identifying the correct statistical definition.

Question 4

This question was a simple routine *Knowledge and procedures* question (1 mark).



What feature of the graph shown above does the interval [2, 8] describe?

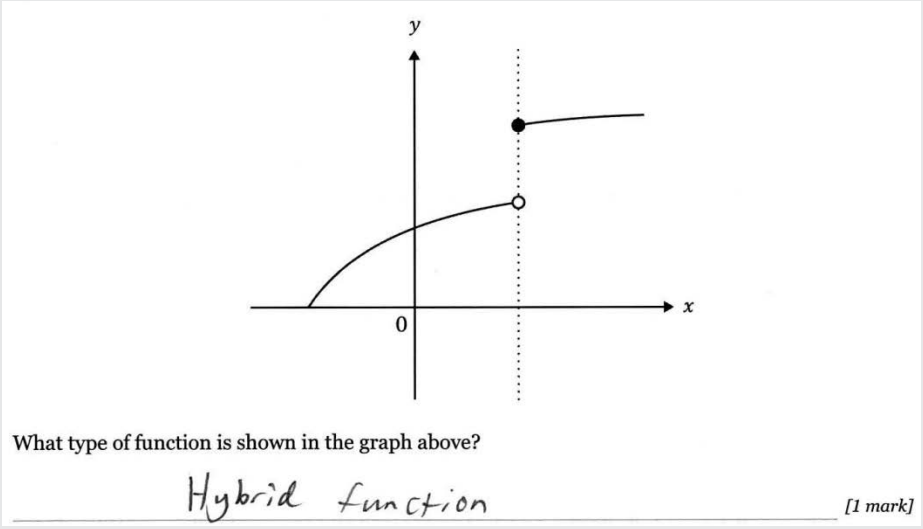
~~[illegible]~~. The range [1 mark]

Students were presented with a sketch of a function constrained to part of the domain. They were required to recognise that a stated interval, given in interval notation, referred to the y-values of the function or the range of the function. A single-word response such as 'range' was expected.

Approximately 70% of students were not able to recognise the interval [2, 8] as being the range of the sketched function. Instead, many different responses were provided such as 'maximum', 'minimum' or 'y-intercept'.

Question 5

This question was a simple routine *Knowledge and procedures* question (1 mark).



Students were presented with a sketch graph depicting a function that differed in two sections of the domain. The sketch also showed a discontinuity located at one point in the domain. The students were required to recall the appropriate function definition that was described by the information shown in the sketch. A single-word response such as ‘hybrid’ or ‘discontinuous’ was expected.

This was a challenging question for students to answer correctly. Many did not recognise the break in the function shown as a discontinuity, or a hybrid function over different parts of the domain. Approximately 70% of students did not respond correctly to this question.

Section B

Question 1

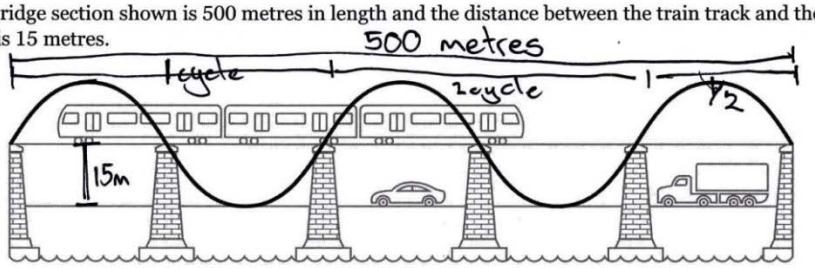
Question 1a was a simple non-routine *Knowledge and procedures* item (1 mark).

Question 1b was a simple non-routine *Knowledge and procedures* and *Communication and justification* question (2 marks and 1 mark respectively).

The design of a two-level bridge frame can be modelled by a sine wave as shown.

The top level is a train track. The lower level is a road for cars and trucks.

The bridge section shown is 500 metres in length and the distance between the train track and the road is 15 metres.



Not to scale

a) State the amplitude of the sine wave used in the design.

15 metre amplitude [1 mark]

b) Determine the period of the sine wave used in the design.

period → a full cycle

in this bridge 2.5 cycles are completed

∴ to find distance or period of one cycle

⇒ 500m ÷ 2.5 cycles

⇒ 200 metres/cycle

∴ 1 cycle/period is 200 m in length

period = 200 metres [3 marks]

Students were required to recall and apply the definitions of amplitude and period to a periodic function presented in the life-related situation of a bridge design. They had to engage with the bridge dimensions provided, recall the definition of amplitude to obtain 15 metres, and use simple mathematical reasoning to determine the period of the sine wave as 200 metres.

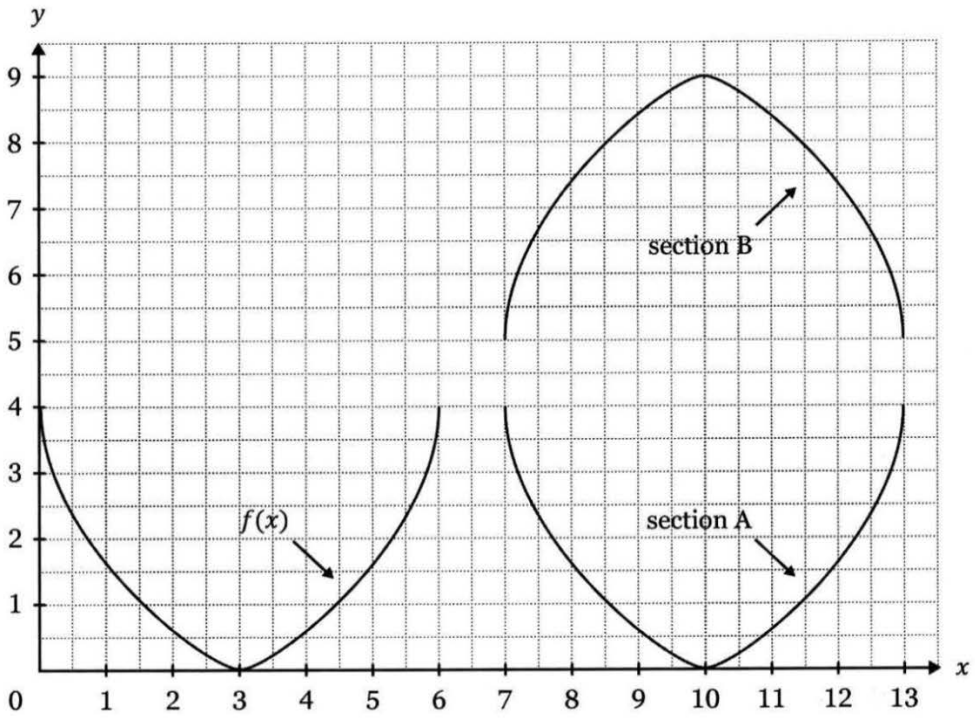
Typically, students were able to engage with both parts of this question. Approximately 30% of students achieved full *KAP* marks. Approximately 60% of students achieved the *CAJ* mark for providing appropriate reasoning to support the period value that was obtained.

Question 2

Question 2a was a simple routine *Knowledge and procedures* and *Communication and justification* question (1 mark each).

Question 2b was a complex routine *Knowledge and procedures* and *Communication and justification* question (2 marks and 1 mark respectively).

A function $f(x)$ defined over the domain $0 \leq x \leq 6$ has been used to form section A and section B shown below.



Transformation notation such as $f(x + a)$, $f(x) + a$, $af(x)$, $f(ax)$ with appropriate values of a can be used to change the position or shape of a function.

a) Using transformation notation, write the function that defines section A.

$f(x-7)$

[As T.P of $(3,0) \rightarrow (10,0)$ = ~~10-7~~ $10-3 = 7$ units right] *[2 marks]*

b) Using transformation notation, write the function that defines section B.

$f(x)$ → Reflected in x-axis

$= -f(x)$

Translated 7 units right [T.P. of $(3,0) \rightarrow (10,9)$]

$= -f(x-7)$

Translated 9 units up [T.P. of $(3,0) \rightarrow (10,9)$] *[3 marks]*

$= -f(x-7)+9$

In Question 2a, students were required to recall and apply the rules of function transformations related to a horizontal translation and to define it using appropriate transformation notation. The response $f(x-7)$ was required.

In Question 2b, students had to use numerical and spatial sense to interpret and describe multiple function transformations that involved both positive and negative values. The students were instructed to use transformation notation to translate the information presented as a labelled sketch into a symbolic representation. The correct response was $-f(x-7)+9$.

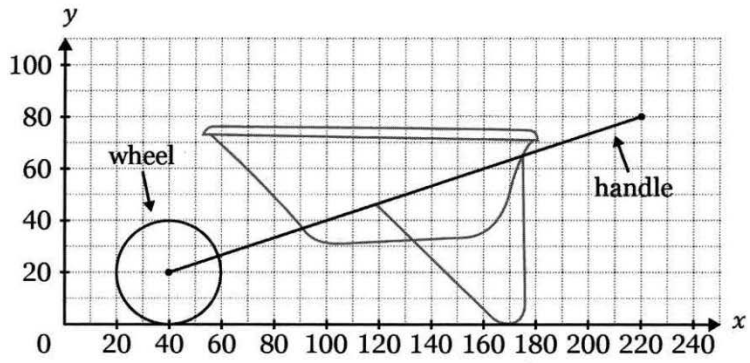
Although recognising the correct transformations involved, often students used transformation notation incorrectly. Common errors were $f(x+7)$ in Question 2a and the incorrect use of the negative sign to describe the reflection in the x-axis, often using $f-(x)$ instead of $-f(x)$, in Question 2b. Approximately 40% of the students achieved between 1 and 3 *KAP* marks for this question.

Question 3

Question 3a was a simple routine *Knowledge and procedures* and *Communication and justification* question (2 marks and 1 mark respectively).

Question 3b was a simple routine *Knowledge and procedures* and *Communication and justification* question (4 marks and 2 marks respectively).

A wheelbarrow is designed using relations and functions. All values on the grid are in centimetres.



a) State the domain and the range of the relation used to represent the wheel.

Domain: $20 \leq x \leq 60$

Range: $0 \leq y \leq 40$ *[3 marks]*

b) What is the equation of the straight line used to represent the handle?

Express your answer in the form $y = mx + c$.

Gradient:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Sub in points (220, 80) and (40, 20)

$m = \frac{20 - 80}{40 - 220}$

$= \frac{-60}{-180}$ 3

$= \frac{1}{3}$

$y = \frac{1}{3}x + c$

Sub in point (40, 20) to find 'c'

$20 = (\frac{1}{3} \times 40) + c$

$20 = \frac{40}{3} + c$

$c = 20 - \frac{40}{3}$

$c = \frac{60}{3} - \frac{40}{3}$

$c = \frac{20}{3}$

∴ Equation of straight line representing handle is ~~$y = \frac{20}{3}$~~ $y = \frac{1}{3}x + \frac{20}{3}$ *[6 marks]*

$y = \frac{1}{3}x + \frac{20}{3}$

In Question 3a, students were required to apply the definitions of domain and range to a circular graph presented on a labelled grid. The domain and range were expected to be stated using appropriate mathematical symbols. Acceptable responses were Domain [20, 60] and Range [0, 40].

Question 3b required students to recall and apply the rules and procedures needed to develop the equation of a straight line, which was presented in a life-related situation on a labelled grid. The expected line equation was $y = \frac{1}{3}x + \frac{20}{3}$.

Students were able to identify the domain and range of the wheel and work towards finding the equation of the straight line. 82% of students found determining the equation of the straight line without error challenging, due to the need to manipulate and simplify fractional values. 85% of students achieved some *KAP* marks for this question, while only 15% of students were awarded full *KAP* marks.

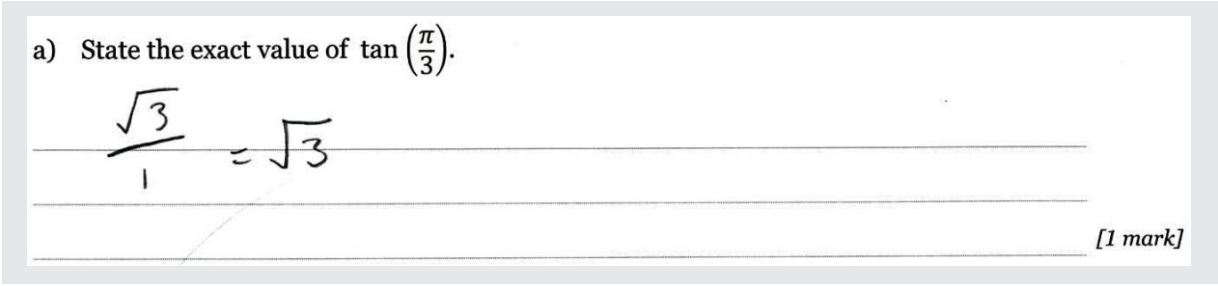
Question 4

Question 4a was a simple routine *Knowledge and procedures* question (1 mark).

Question 4b was a simple routine *Knowledge and procedures* and *Communication and justification* question (2 marks and 1 mark respectively).

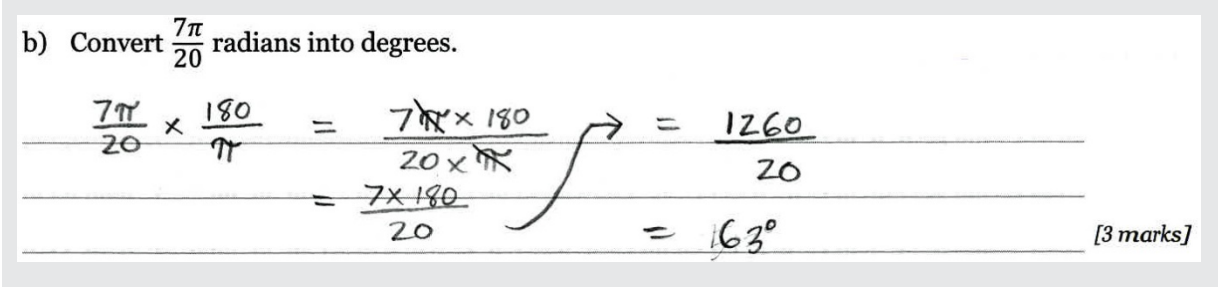
Question 4c was a simple routine *Knowledge and procedures* and *Communication and justification* question (3 marks and 1 mark respectively).

Question 4d was a complex routine *Knowledge and procedures* and *Communication and justification* question (4 marks and 2 marks respectively).


a) State the exact value of $\tan\left(\frac{\pi}{3}\right)$.

$\frac{\sqrt{3}}{1} = \sqrt{3}$

[1 mark]


b) Convert $\frac{7\pi}{20}$ radians into degrees.

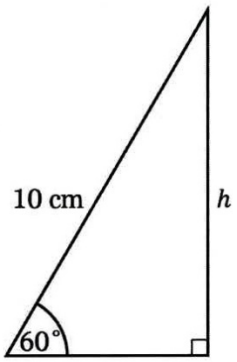
$\frac{7\pi}{20} \times \frac{180}{\pi} = \frac{7\pi \times 180}{20 \times \pi} = \frac{1260}{20}$

$= \frac{7 \times 180}{20}$

$= 63°$

[3 marks]

c) Determine the exact value of h shown in the triangle below.



Not to scale

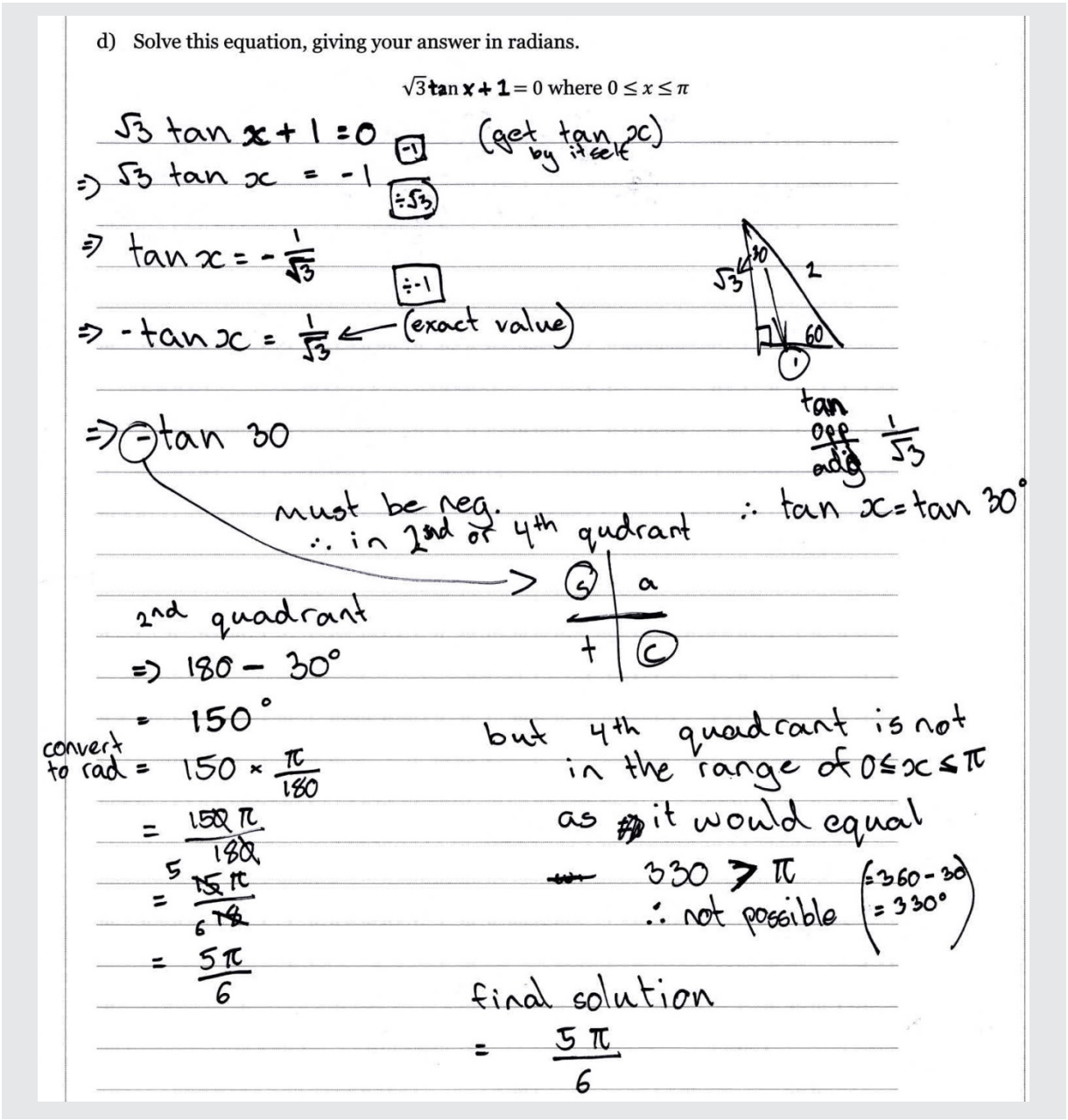
$\sin 60° = h/10$

$\sqrt{3}/2 = h/10$

$\frac{10\sqrt{3}}{2} = \frac{2h}{2}$

$5\sqrt{3}\ \text{cm} = h$

[4 marks]



In Question 4a, students were required to state the exact value of the tangent ratio for a specified angle that was given in radian measure. 35% of students achieved success stating the required exact value of $\sqrt{3}$.

In Question 4b, students were required to access and use the procedure needed to convert radians to degrees. Numerical calculations were required to obtain the angle. Students used correct procedure and calculations to obtain the required angle of $63°$.

In Question 4c, students were required to select and apply definitions to determine the length of an unknown side in a right-angled triangle. Algebraic facility and numerical calculations were needed to express the length as an exact value.

Typically, students could engage successfully with this question, recognising the need to use the sine ratio and/or appropriate exact values. However, 77% of students could not consistently complete the simplification of the fractional exact value correctly. The expected response was $5\sqrt{3}$ cm.

In Question 4d, students were required to solve a trigonometric equation within a given domain. This involved the algebraic facility to rearrange an equation, the recall and application of trigonometric definitions and numerical calculations to determine an angle in the appropriate quadrant.

Students achieved some success with equation manipulation and determining an angular value. However, 12% of students linked all of the required algebraic and domain considerations to complete the question accurately to obtain the solution to the equation of $x = \frac{5\pi}{6}$.

Question 5

This question was a simple routine *Modelling and problem-solving* and *Communication and justification* question (4 marks and 2 marks respectively).

A researcher asked two groups of teenagers to record the number of text messages they received or sent in a 24-hour period.

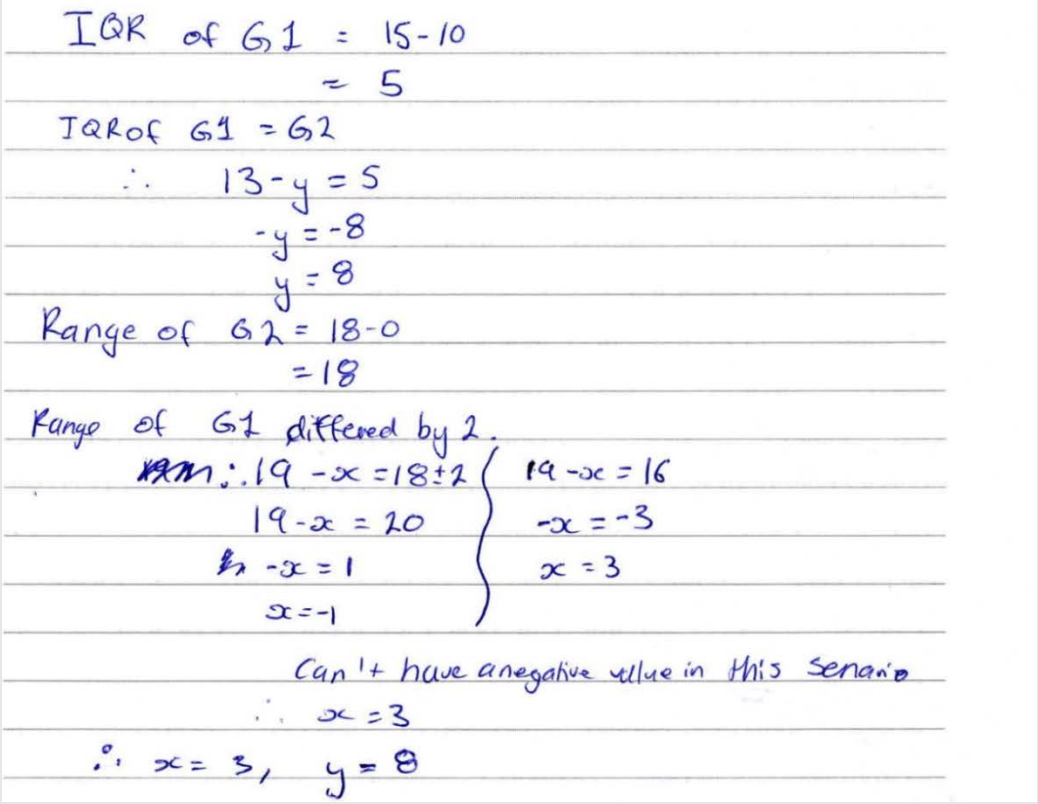
The results for each group are given here as five number summaries.

Group 1	min = x	$Q_1 = 10$	med = 12	$Q_3 = 15$	max = 19
Group 2	min = 0	$Q_1 = y$	med = 9	$Q_3 = 13$	max = 18

The researcher found:

- the interquartile ranges of both groups were equal
- the ranges of the groups differed by 2.

Determine the values of x and y.



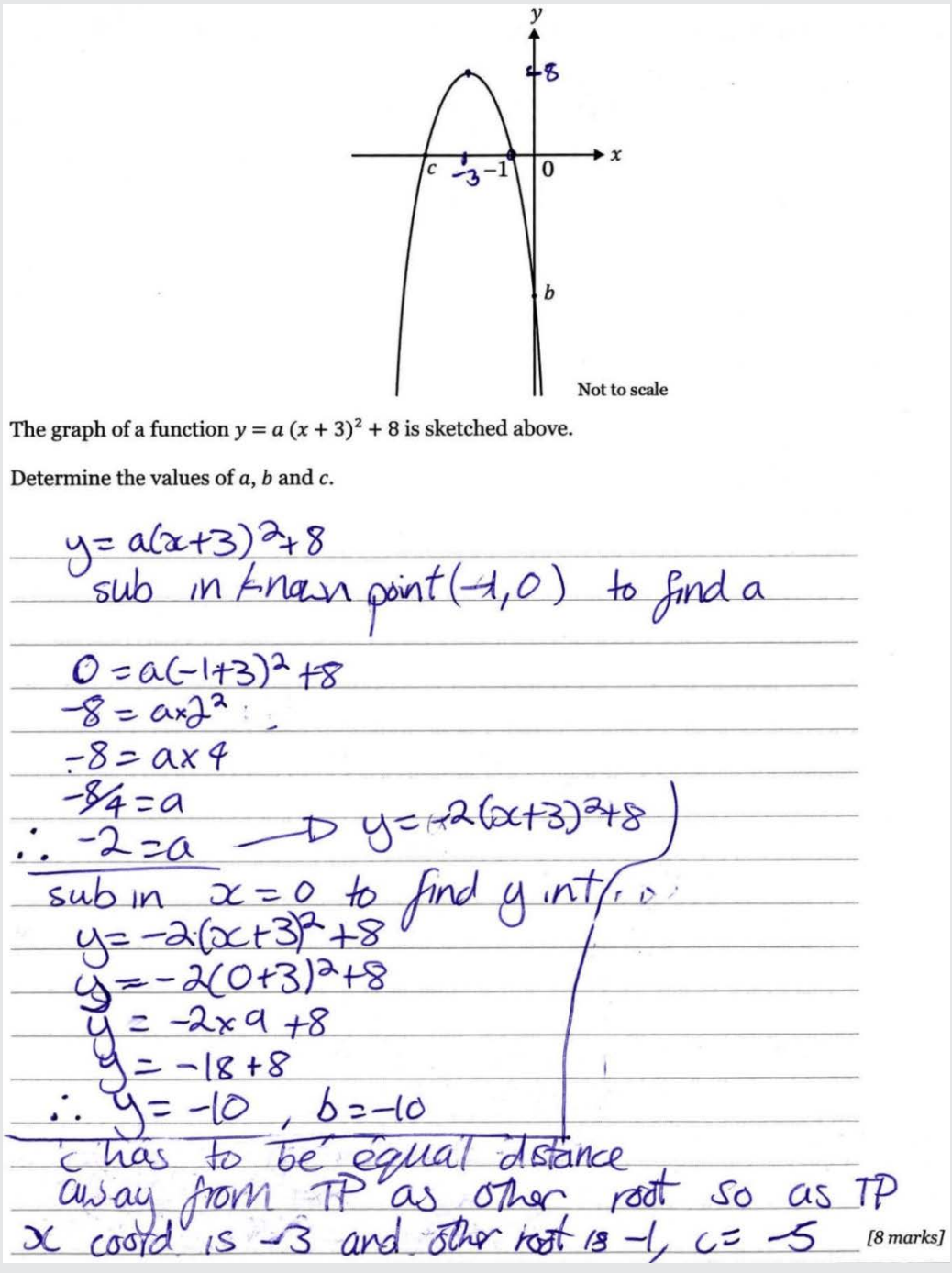
Question 5 involved the use of problem-solving strategies and reasoning to determine unknown values by comparing two five number summaries that had the same interquartile range, and range values that differed by 2.

Students were required to compare the interquartile range of each group to determine $y = 8$. Group 1 could have two possible range values, 16 and 20. However, only a range of 16 resulted in a feasible (positive) value for x. The expected value was $x = 3$.

Overall, 55% of students achieved full *KAP* marks for this question and 65% were awarded the full *CAJ* marks.

Question 6

This question was a simple routine *Modelling and problem-solving* and *Communication and justification* question (5 marks and 3 marks respectively).



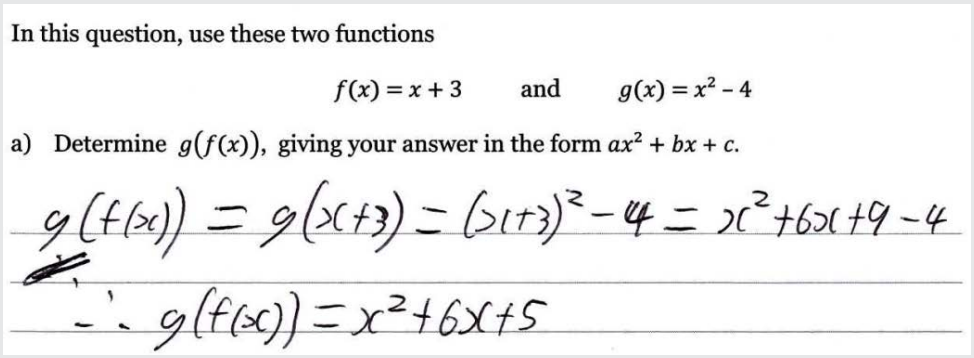
Question 6 required students to determine an unknown parameter in a quadratic polynomial and two unknown axes intercepts that were identified on a sketch graph. This question offered students opportunities to use problem-solving strategies and mathematical reasoning to develop coherent sequences within the response. The expected values were $a = -2,\ b = -10,\ c = -5$.

13% of students connected the concepts required to determine all three unknowns correctly. Just over 50% of students achieved partial to full success with *MAP* and *CAJ* for this question.

Question 7

Question 7a was a complex routine *Knowledge and procedures* and Communication and justification question (3 marks and 1 mark respectively).

Question 7b was a complex non-routine *Modelling and problem-solving* and *Communication and justification* question (3 marks and 1 mark respectively).

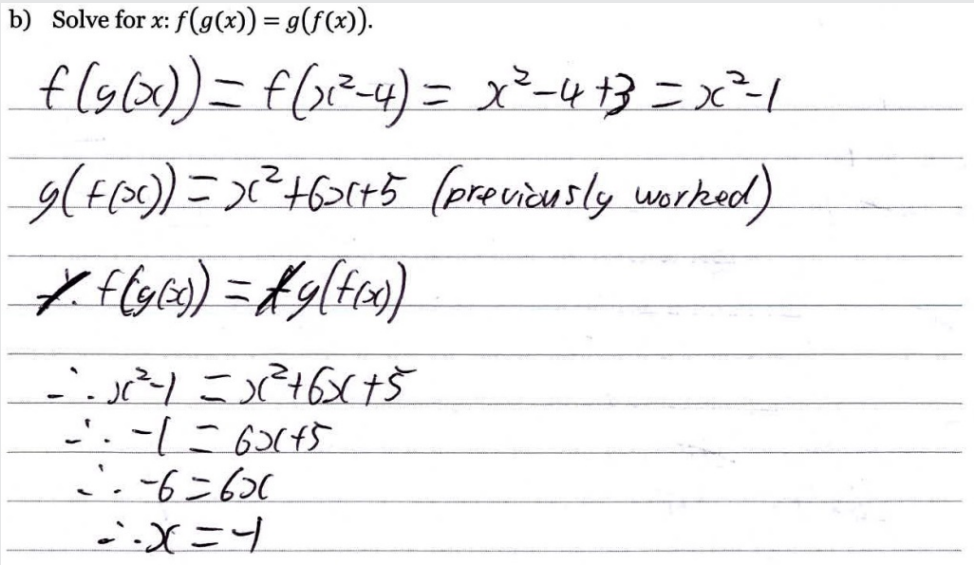

In this question, use these two functions

$f(x) = x + 3$ and $g(x) = x^2 - 4$

a) Determine $g(f(x))$, giving your answer in the form $ax^2 + bx + c$.

$g(f(x)) = g(x+3) = (x+3)^2 - 4 = x^2 + 6x + 9 - 4$

$\therefore g(f(x)) = x^2 + 6x + 5$


b) Solve for x: $f(g(x)) = g(f(x))$.

$f(g(x)) = f(x^2 - 4) = x^2 - 4 + 3 = x^2 - 1$

$g(f(x)) = x^2 + 6x + 5$ (previously worked)

$\therefore f(g(x)) = g(f(x))$

$\therefore x^2 - 1 = x^2 + 6x + 5$

$\therefore -1 = 6x + 5$

$\therefore -6 = 6x$

$\therefore x = -1$

In Question 7a, students were required to recall and apply rules for function notation and then use algebraic facility to produce an expression in quadratic form using appropriate mathematical symbols. The simplified answer to this question was $x^2 + 6x + 5$.

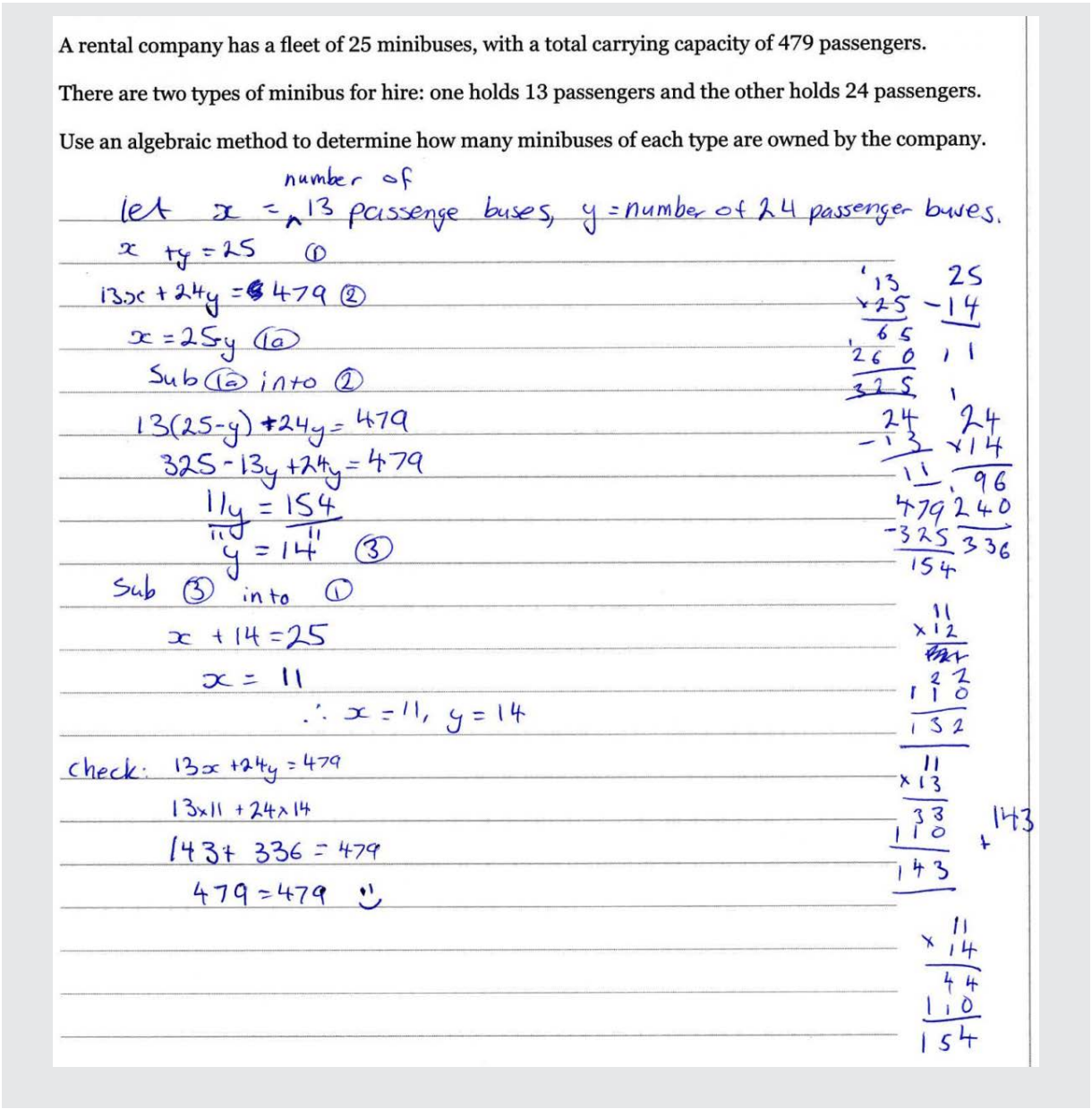
Approximately 45% of students understood how to use composite function notation and obtain the quadratic expression result. However, about 35% of students received no *KAP* marks.

Question 7b offered opportunities to use problem-solving strategies and mathematical reasoning to develop a solution to an equation resulting from two quadratic composite functions. The solution to the equation was $x = -1$.

Approximately 30% of students were able to equate two equations and link all aspects of the question to arrive at the correct solution.

Question 8

This question was a complex routine *Modelling and problem-solving* and *Communication and justification* question (4 marks and 2 marks respectively).



In Question 8, students were required to interpret and analyse a life-related problem, translate the given numerical information into two equations and then use an algebraic method to solve simultaneous equations. The required equations were of the form $a + b = 25$ and $13a + 24b = 479$. The solutions were $a = 11$ and $b = 14$.

Approximately 50% of students were able to engage with this question and achieve either partial success or the correct solution.

Section C

Multiple choice

Questions 1–5 were simple routine *Knowledge and procedures* questions.

Questions 6–10 were simple routine *Modelling and problem-solving* questions.

All multiple-choice questions were worth one mark each.

Question number	Students are required to demonstrate understanding of:	Correct answer	% correct responses
1	• using a procedure to convert an angle in degrees into radians.	A	86
2	• using technology (if desired) set in the context of function notation.	D	47
3	• recalling the definition of a discrete variable.	B	56
4	• recalling the definition of a function.	C	63
5	• recognising the shape of a cubic function.	A	62
6	• using simple problem-solving strategies while interpreting information contained in a bar chart.	C	72
7	• using simple problem-solving strategies to recognise the gradient ratio within a diagram labelled with dimensions.	A	79
8	• interpreting a stem-and-leaf plot of data to determine the matching boxplot.	A	79
9	• rearranging the cosine rule to the form where the subject of the rule was $\cos\theta$.	B	56
10	• interpreting numerical data to determine a mean value.	C	83

Section D

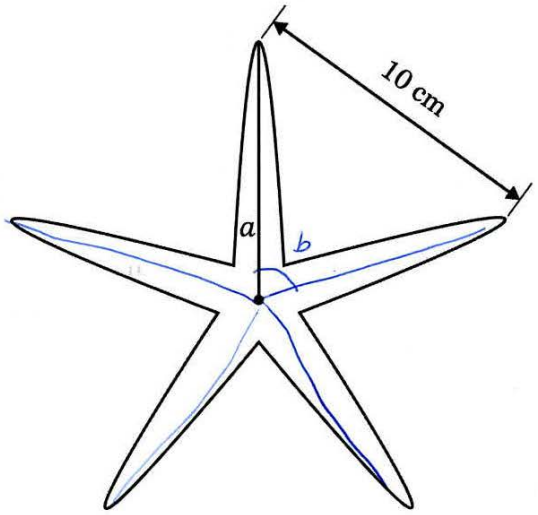
Question 1

This question was a simple non-routine *Knowledge and procedures* and *Communication and justification* question (3 marks and 2 marks respectively).

A research student measured the tip-to-tip length between the arms of a starfish to be 10 cm, as shown in the diagram.

- All angles between the arms were equal.
- All arms were equal in length.

Starfish



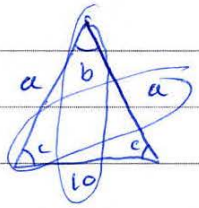
Not to scale

Use the sine rule to calculate the length of one arm, a, from the centre of the starfish to the tip.

5 angles all equal

$\therefore 360 \div 5$

$= 72°$ angle b.



$\angle$ in $\triangle$ add to 180

$\therefore 180 = 72 + 2c$

$2c = 108$

$c = 54$

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin 54} = \frac{10}{\sin 72}$

$a = \frac{10}{\sin 72} \times \sin 54$

$a = 8.51$ cm.

In Question 1, students were instructed to use the sine rule to calculate an unknown length, in a life-related situation.

Once a central angle of 72º was identified, students were able to complete this question successfully. A common error was to misinterpret the starfish diagram as consisting of equilateral triangles, resulting in the length of an arm to be 10 cm.

Students were required to determine each central angle to be 72º, the angle at each tip to be 54º and to use the sine rule to calculate an arm length of 8.51 cm.

Approximately 30% of all students achieved full *KAP* marks for this question.

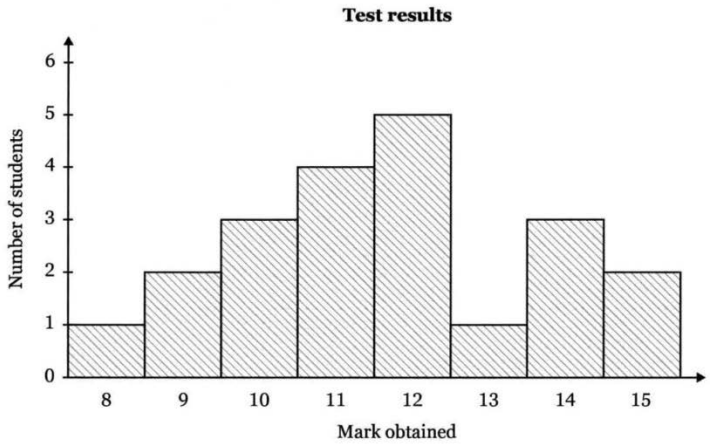
Question 2

Question 2a was a simple routine *Knowledge and procedures* and *Communication and justification* question (1 mark each).

Question 2b was a simple routine *Knowledge and procedures* and *Communication and justification* question (3 marks and 2 marks respectively).

Question 2c was a simple non-routine *Knowledge and procedures* and *Communication and justification* question (3 marks and 2 marks respectively).

The histogram shows the results of a class test. No fractional marks were awarded.

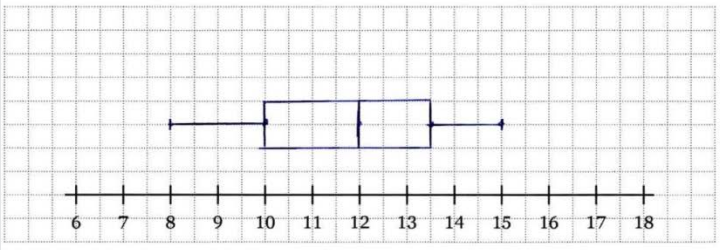


a) How many students were in this class?

1 + 2 + 3 + 4 + 5 + 1 + 3 + 2
= 21 students

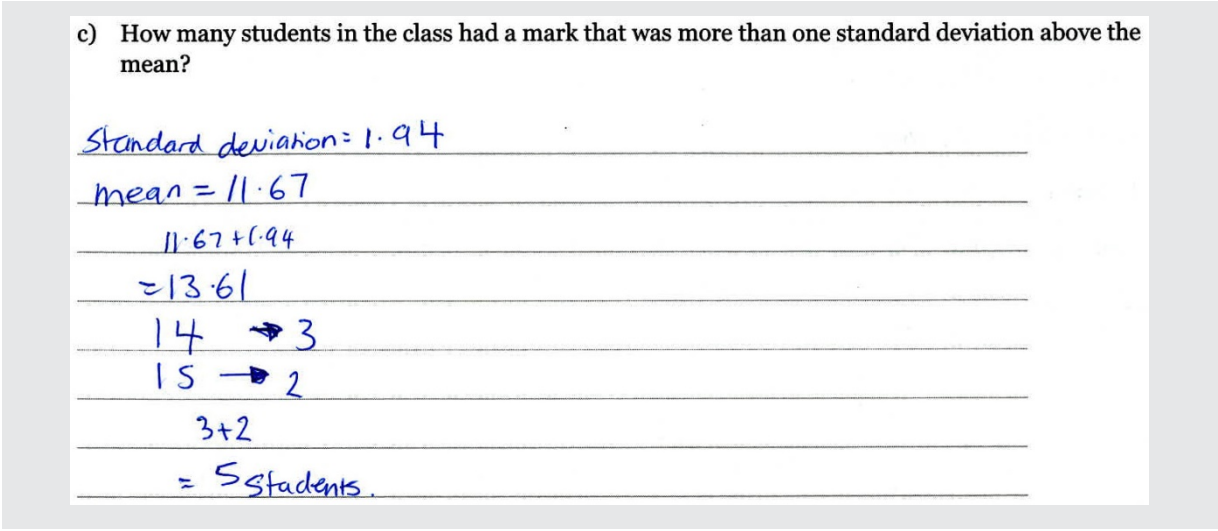
[2 marks]

b) Draw the boxplot (box-and-whisker plot) using the information in the histogram.



[5 marks]

max = 15 med = 12 min = 8
Q3 = 13.5 Q1 = 10


c) How many students in the class had a mark that was more than one standard deviation above the mean?

Standard deviation = 1.94
mean = 11.67
11.67 + 1.94
= 13.61
14 → 3
15 → 2
3 + 2
= 5 students.

In the three parts of Question 2, students were required to apply statistical definitions and procedures to information presented in a histogram. Technology was available to produce a boxplot drawn to scale on a grid and to use the mean and standard deviation of the histogram data to answer a question about students in this class.

Question 2a was completed successfully by 94% of students, who correctly determined 21 students as the answer. However, often no marks were awarded for *CAJ*, as reasoning was not provided to explain how this number of students was determined.

49% of students were able to correctly determine and draw the boxplot of the histogram data in Question 2b. The five number summary values were 8, 10, 12, 13.5, 15.

Students effectively used the mean (11.67) and standard deviation (1.94) to answer Question 2c, despite some occasional transcription errors of the data being evident. The correct answer was 5 students.

Overall, approximately 75% of students achieved 3 or 4 *KAP* marks out of a total of 7 marks.

Question 3

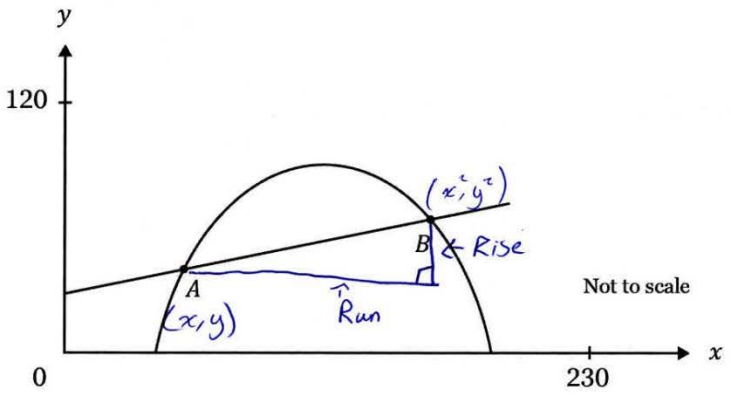
This question was a simple routine *Modelling and problem-solving* and *Communication and justification* question (4 marks each).

A line and parabola are sketched below. A and B are the points of intersection.

The line has the equation $y = \frac{1}{4}x + 22$

The parabola has the equation $y = -\frac{1}{60}x^2 + 4x - 160$

Calculate the straight-line distance between the points A and B. No units are required in your answer.



$y_1 = \frac{1}{4}x + 22$

$y_2 = -\frac{1}{60}x^2 + 4x - 160$

Find intersecting points use simultaneous equations on calc.

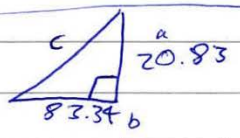
$y_1 = \frac{1}{4}x + 22$

$y_2 = -\frac{1}{60}x^2 + 4x - 160$ → Graph

→ 2nd → Calc → 5: intersect

A = $x = 70.83$ B = $x_2 = 154.17$

$y = 39.71$ $y_2 = 60.54$



(Rise) $y_2 - y_1 = 60.54 - 39.71$

$= 20.83$

$x_2 - x_1 = 159.17 - 70.83$

$= 83.34$

$c^2 = a^2 + b^2$

$c^2 = 70.83^2 + 83.34^2$

$c^2 = 7379.4945$

$c = \sqrt{7379.4945}$

$c = 85.90$

[8 marks]

Students were required to determine the straight-line distance between the two points of intersection between a line and a parabola that were presented both as equations and as a sketch graph.

The question provided opportunities for students to use problem-solving strategies to interpret and analyse the problem. Once the two points of intersection were obtained, the students then

needed to decide on using either Pythagoras' theorem or the Distance formula as a strategy required to progress to the correct answer.

The use of a graphics calculator proved to be an efficient method to solve this question. Students who opted for an algebraic approach to find the points of intersection usually produced manipulative errors.

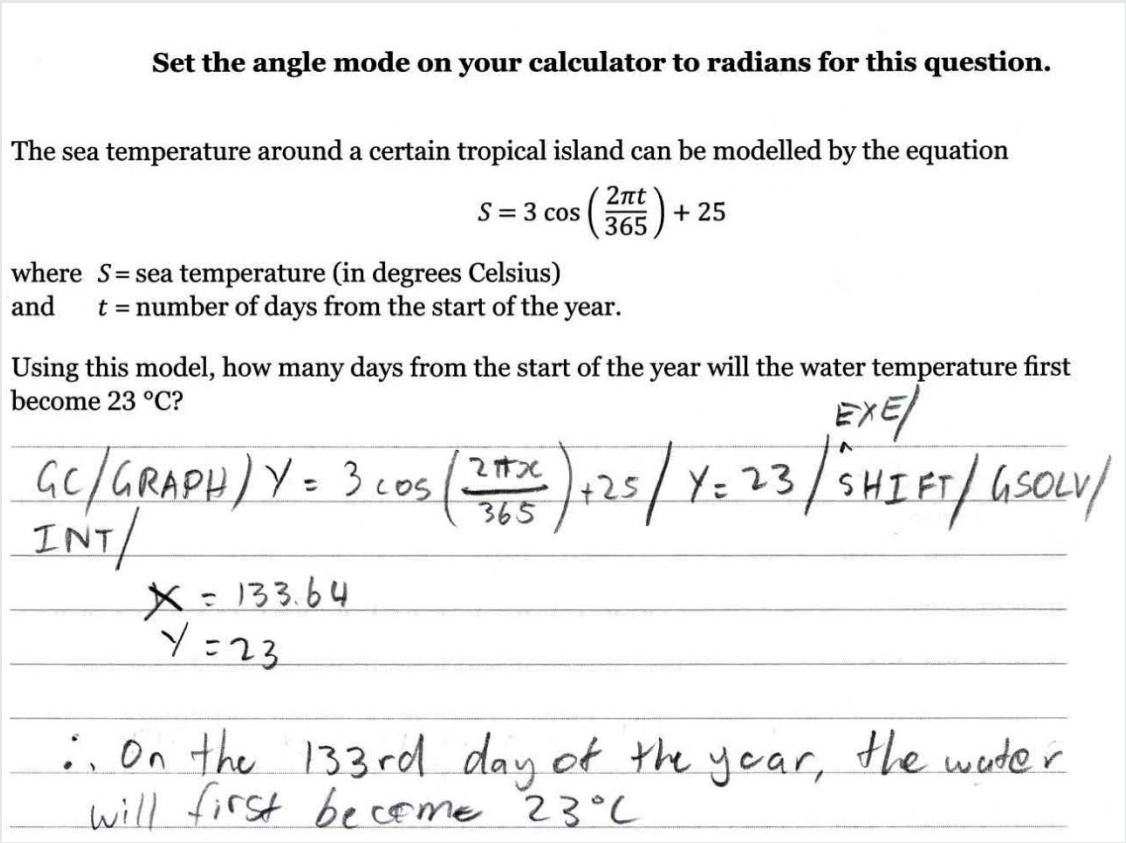
A common mistake was made in interpreting the distance required as the distance between the x coordinate values alone.

The points of intersection were A (70.83, 39.71) and B (154.17, 60.54). The rise (20.83) and run (83.34) when substituted into either Pythagoras' theorem or the Distance formula, produce a straight-line distance of 85.90 cm.

Approximately 70% of all students were unable to achieve any of the *MAP* marks allocated for this question.

Question 4

This question was a simple routine *Modelling and problem-solving* and *Communication and justification* question (2 marks each).



Set the angle mode on your calculator to radians for this question.

The sea temperature around a certain tropical island can be modelled by the equation

$$S = 3\cos\left(\frac{2\pi t}{365}\right) + 25$$

where S = sea temperature (in degrees Celsius)
and t = number of days from the start of the year.

Using this model, how many days from the start of the year will the water temperature first become 23 °C?

GC/GRAPH/ $Y = 3\cos\left(\frac{2\pi x}{365}\right)+25$ / $Y=23$ / SHIFT/ GSOLV/ INT/ EXE/

$x = 133.64$

$Y = 23$

∴ On the 133rd day of the year, the water will first become 23°C

Question 4 offered students the opportunity to interpret mathematical terminology presented as a trigonometric model of a life-related situation and to decide on a problem-solving strategy to use.

When a graphics calculator was used, this question was usually answered correctly. However, many students attempted to solve the trigonometric equation using algebraic methods. This algebraic approach often did not allow students to complete the question, with students progressing only part-way through the algebraic manipulation required.

Students were expected to substitute 23 into the S variable of the equation, resulting in a solution for t of 133.64. This was to be interpreted as either on the 133rd day or 134th day as the first whole day when the required temperature was reached.

Approximately 65% of students achieved part-marks or better for both *MAP* and *CAJ* in this question.

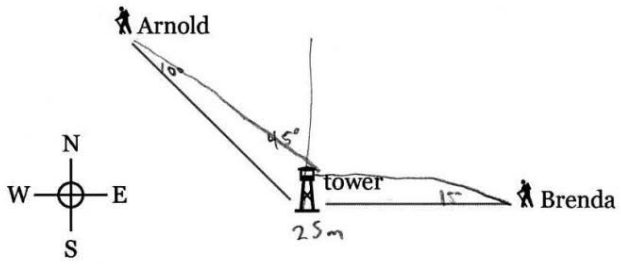
Question 5

This question was a complex routine *Modelling and problem-solving* and *Communication and justification* question (4 marks and 2 marks respectively).

Arnold and Brenda are walking in different locations in a flat area of land. They are both able to see the top of a 25-metre high tower.

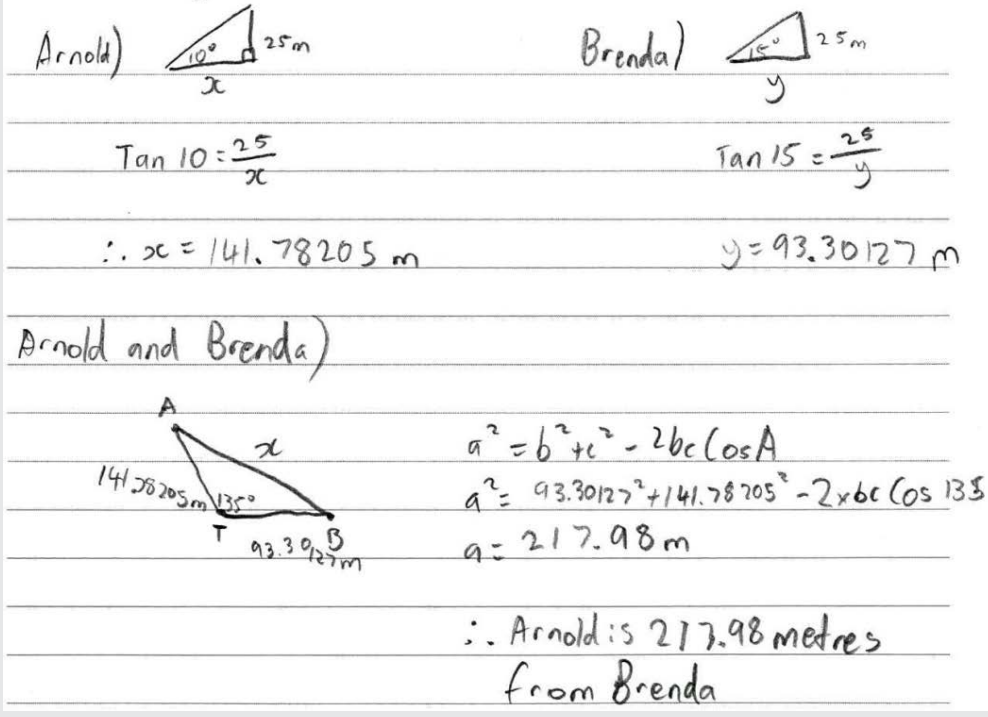
Arnold is north-west (N45°W) of the tower and he measures the angle of elevation to the top of the tower as 10°.

Brenda is east of the tower and she measures the angle of elevation to the top of the tower as 15°.



Not to scale

Calculate the straight-line distance between Arnold and Brenda at this moment.



Students were required to interpret and analyse a three-dimensional situation, using problem-solving strategies that involved right-angled triangle trigonometry and the cosine rule. Mathematical reasoning was required to develop a logical sequence linking the two- and three-dimensional aspects of the problem. The key aspects in an expected response for this question were the trigonometric calculation of distances of Arnold and Brenda from the tower (141.78 m

and 93.30 m respectively), an angle of 135º between the two walkers, and the use of the cosine rule to calculate the distance between them of 217.98 m.

50% of students achieved at least partial success with this question, with approximately 15% of all students receiving full *MAP* marks. Occasionally students did not change calculators back into degree mode from the previous question that used radians. Often 155º was used in the calculation instead of the 135º required. Students frequently attempted to use Pythagoras' theorem instead of the cosine rule. Question 5 was also prone to students rounding off answers continually throughout the calculation stages, resulting in a final distance that was inaccurate.

Question 6

Question 6a was a simple routine *Modelling and problem-solving* and *Communication and justification* question (2 marks each).

Question 6b was a simple routine *Modelling and problem-solving* and *Communication and justification* question (2 marks each).

Question 6c was a complex non-routine *Modelling and problem-solving* and *Communication and justification* question (5 marks and 2 marks respectively).

There are many costs involved in the maintenance of Australian forests, particularly near housing and residential areas. One of these maintenance costs is controlled burning.

The data in the table shows the maintenance cost per hectare ($/ha) including controlled burning in forest areas.

Area burnt, x (ha)	190	170	380	q
Maintenance cost per hectare including burning, y ($/ha)	5880	p	3505	4505

a) Form a linear mathematical model describing the data in the table above.

GC/STAT/List 1 → 190, 380/List 2 → 5880, 3505/GRAPH/CALC/X/ax+b/

$Y = -12.5x + 8255$

b) Use your model to determine the missing data values, p and q, in the table above.

$Y = -12.5x + 8255$ if $x = q$ and $y = 4505$

if $x = 170$ and $p = y$

$p = -12.5 \times 170 + 8255$

$p = \$6130/ha$

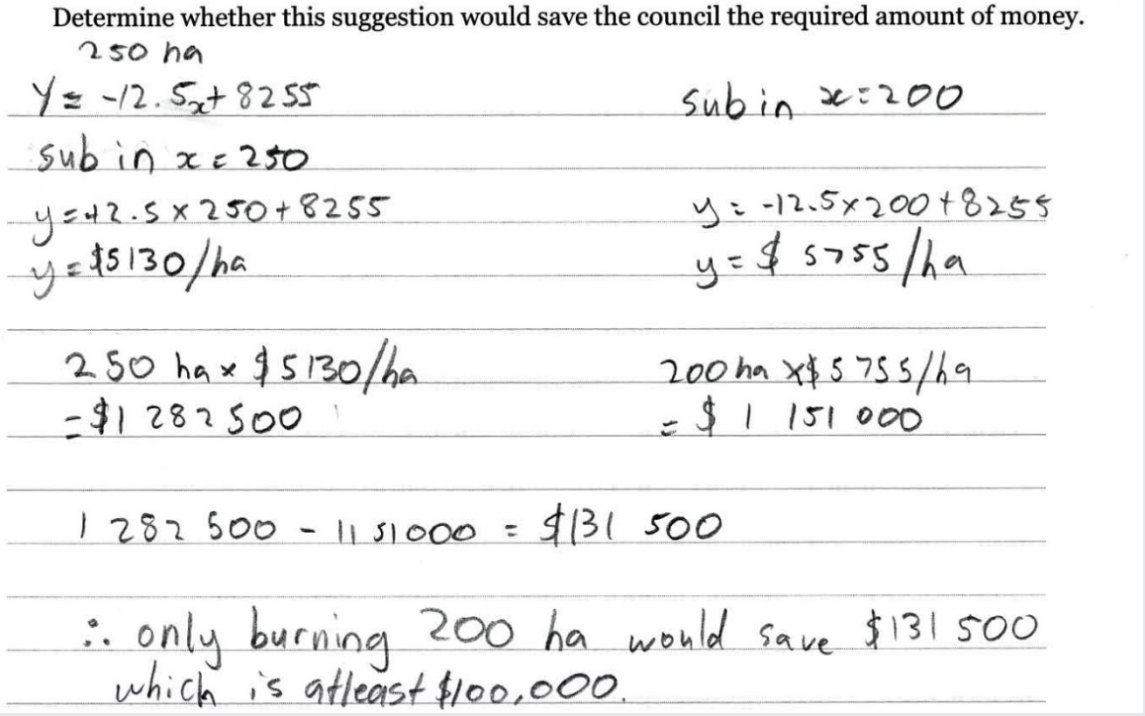
$4505 = -12.5q + 8255$

$q = 300$ ha

c) In its forest maintenance program, a council is planning to burn 250 ha of forest.

The council needs to cut costs and save at least \$100 000 in its forest maintenance spending. A suggestion has been made to burn 200 ha of forest instead of 250 ha.

Determine whether this suggestion would save the council the required amount of money.



In Question 6, students were required to use tabular data of a life-related situation to synthesise the linear mathematical model given by the equation $y = -12.5x + 8255$. By applying the model, students were required to justify results, $p = 6130$ when $x = 170$ and $q = 300$ when $y = 4505$ and a decision made based on cost-saving issues. The total cost-saving made by the suggestion was \$131 500, according to the model. Therefore, the decision to be made is 'yes', because this figure saves more than the required \$100 000.

In Question 6a, the use of a graphics calculator would be an efficient way to determine the linear model. However, students who chose to use algebraic methods instead, took more time and were prone to calculation errors.

Once a model was obtained in Question 6a, the students usually achieved success with Question 6b, with occasional minor calculation errors present.

Those students that obtained a linear model in Question 6a were able to make some progress in Question 6c. These students were usually able to substitute 200 ha and 250 ha correctly into the linear model to obtain costs per hectare. However, few students progressed with this information to correctly determine the financial savings that could be achieved by the suggestion made in the question.

Overall, approximately 40% of students received partial or full *MAP* marks for Question 6. It was notable that some students did not attempt this question. Students need to ensure they are managing their time. This question was written as unfamiliar and non-routine, and this may have contributed to non-responses.

Recommendations and guidelines

- *Students need to have a proficient understanding of the functionality of a graphics calculator (non-CAS) for a technology-active assessment.*

  In Paper 2 — technology active, students benefit from identifying opportunities to use graphics calculator features. Apart from using a calculator for the obvious statistical calculations that arise when dealing with data (Question 2), and trigonometric calculations involving angles (Questions 1 and 5), students need to be familiar with using a graphics calculator to solve simultaneous equations (Question 3), analyse graphical situations (Question 4) and produce mathematical models from data (Question 6).

  In this trial, students often chose to use an algebraic approach to answer Questions 3, 4 and 6a. Using algebraic procedures in the technology-active paper is time-consuming, increases the likelihood of making careless errors and is unnecessarily challenging when the values in a question are not easy-to-use numbers (such as integers or simple fractions). Effective use of a graphics calculator enables more efficient use of time, especially in an examination situation, and can provide more reliable answers. To prepare for a technology-active paper, students should be encouraged to explore the functionality of a graphics calculator and practise using it whenever possible.

- *Students should make use of the accuracy provided by using a calculator. It is only the final answer to a question that should be rounded off.*

  Paper 2 in this trial instructed students to state all final answers to two decimal places. This refers to the final answers only. Students should not round calculations during the stages of answering a question, as this can result in a compounded rounding error, leading to a final answer outside the appropriate range deemed to be correct for that question. Students should be explicitly taught to keep decimal places alive in the calculator (despite maybe writing down shorter values in the written work) and round off to the required decimal places only for the final answer.

- *Students must communicate how they used their calculator to find a solution with an adequate written record*.

  Transferring a sketch of a graphical method used to solve an equation to their paper is an example of how students must communicate when using a calculator to find a solution. In Paper 2 — technology active, where a calculator was used in Questions 2, 3, 4 and 6, there should be some written communication to describe how numerical values were obtained.

- *Students should check their calculations.*

  Initial errors made in a question were usually not penalised more than once. Provided the subsequent mathematics was correct, follow-through errors were allowed in this trial. However, checking calculations is highly recommended if time permits.

- *Students should also consistently practise checking the reasonableness of their results.*

  They should be aware that there are often a variety of methods and checks that can be used. For example:

  - recognising that a negative length for a distance is not appropriate
  - recognising that points of intersection have two coordinates (x and y)
  - understanding that an answer may be incorrect if it has decimals, for example finding the number of buses in Question 8 of Paper 1.

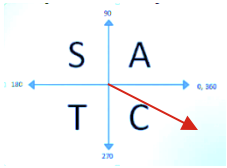
Appendix 1: Instrument-specific standards matrix

	A	B	C	D	E
	The student work has the following characteristics:				
Knowledge and procedures	• recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations	• recall, access, selection of mathematical definitions, rules and procedures in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations	• recall, access, selection of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations	• use of stated rules and procedures in simple situations	• statements of relevant mathematical facts
	• application of mathematical definitions, rules and procedures in routine and non-routine simple tasks, through to routine complex tasks, in life-related and abstract situations	• application of mathematical definitions, rules and procedures in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations	• application of mathematical definitions, rules and procedures in routine, simple life-related or abstract situations		
	• numerical calculations, spatial sense and algebraic facility in routine and non-routine simple tasks through to routine complex tasks, in life-related and abstract situations	• numerical calculations, spatial sense and algebraic facility in routine or non-routine simple tasks, through to routine complex tasks, in either life-related or abstract situations	• numerical calculations, spatial sense and algebraic facility in routine, simple life-related or abstract situations	• numerical sense, spatial sense and/or algebraic facility in routine or simple tasks	
	• appropriate selection and accurate use of technology	• appropriate selection and accurate use of technology	• selection and use of technology	• use of technology	• use of technology
Modelling and problem solving	• use of problem-solving strategies to interpret, clarify and analyse problems to develop responses from routine simple tasks through to non-routine complex tasks in life-related and abstract situations	• use of problem-solving strategies to interpret, clarify and analyse problems to develop responses to routine and non-routine simple tasks through to routine complex tasks in life-related or abstract situations	• use of problem-solving strategies to interpret, clarify and develop responses to routine, simple problems in life-related or abstract situations	• evidence of simple problem-solving strategies in the context of problems	• evidence of simple mathematical procedures
	• use of data to synthesise mathematical models and generation of data from mathematical models in simple through to complex situations	• use of data to synthesise mathematical models in simple situations and generation of data from mathematical models in simple through to complex situations	• use of mathematical models to represent routine, simple situations and generate data	use of given simple mathematical models to generate data.	

	A	B	C	D	E
	The student work has the following characteristics:				
Communication and justification	• appropriate interpretation and use of mathematical terminology, symbols and conventions from simple through to complex and from routine through to non-routine, in life-related and abstract situations	• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple or complex and from routine through to non-routine, in life-related or abstract situations	• appropriate interpretation and use of mathematical terminology, symbols and conventions in simple routine situations	• use of mathematical terminology, symbols or conventions in simple or routine situations	• use of mathematical terminology, symbols or conventions
	• organisation and presentation of information in a variety of representations	• organisation and presentation of information in a variety of representations	• organisation and presentation of information	• presentation of information	• presentation of information
	• analysis and translation of information from one representation to another in life-related and abstract situations from simple through to complex and from routine through to non-routine	• analysis and translation of information from one representation to another in life-related or abstract situations, simple or complex, and from routine through to non-routine	• translation of information from one representation to another in simple routine situations		
	• use of mathematical reasoning to develop coherent, concise and logical sequences within a response from simple through to complex and in life-related and abstract situations using everyday and mathematical language	• use of mathematical reasoning to develop coherent and logical sequences within a response in simple or complex and in life-related or abstract situations using everyday and/or mathematical language	• use of mathematical reasoning to develop sequences within a response in simple routine situations using everyday or mathematical language		
	• coherent, concise and logical justification of procedures, decisions and results	• coherent and logical justification of procedures, decisions and results	• justification of procedures, decisions or results		
	• justification of the reasonableness of results				

Appendix 2: Marking guide

		Simple routine		
Questions A1 to A5		**KAP**	1, 3	**CAJ**	Nil
Knowledge and Procedures				

Model Response	KAP Marking Allocation	CAJ Marking Allocation
QUESTION A1 *quadrant* 4 *or* 4*th quadrant* *or* 4 *or* 4*th* *or a suitable diagram*  	**QUESTION A1** [numerical sense, spatial sense and/or algebraic facility] *any of the various ways of describing quadrant* 4 • 1 mark	
QUESTION A2 cos θ *or cosine* θ *or* cos *or cosine*	**QUESTION A2** [recall, access, selection of mathematical definitions, rules and procedures] *any of the various ways of stating the cosine ratio* • 1 mark	

QUESTION A3 *interquartile range* *or IQR* *or middle* 50%	**QUESTION A3** [statements of relevant mathematical facts] *any of the various ways of stating interquartile range* • 1 mark	
QUESTION A4 *range* *or y values*	**QUESTION A4** [numerical sense, spatial sense and/or algebraic facility] *any of the various ways of stating range* • 1 mark	
QUESTION A5 *discontinuous* *or discontinuous function* *or a discontinuity* *or* *or hybrid* *or mixed* *or piecewise defined* *or piecewise* Note: if more than one response is given to a question in Section A, where one of the responses is incorrect, no mark is to be awarded.	**QUESTION A5** [recall, access, selection of mathematical definitions, rules and procedures] *any appropriate descriptor of the discontinuity feature shown in the graph* *or* *any descriptor of two different functions in different parts of the domain* • 1 mark	
	5 marks	**0 marks**

			Simple non-routine	
Questions B1a) and B1b)	**KAP**	1, 2, 3	**CAJ**	4

Model Response	KAP Marking Allocation	CAJ Marking Allocation
QUESTION B1a) amplitude $= 15\ m$	**QUESTION B1a)** [recall, access, selection of mathematical definitions, rules and procedures] *the value* "15" *is stated, with or without units* • 1 mark	
QUESTION B1b) period $= \frac{500}{2.5}$ $= \frac{5000}{25}$ $= 200\ m$	**QUESTION B1b)** [application of mathematical definitions, rules and procedures] [numerical calculations, spatial sense and algebraic facility] *a division of relevant numerical values is evident* OR *a written description of a valid procedure is evident* • 1 mark *the final value of* "200" *is stated, with or without units* • 1 mark	**QUESTION B1b)** [use of mathematical reasoning to develop coherent and logical sequences within a response] *reasoning is given to support the numerical value for the period, i.e. how the period was obtained (KAP errors may be present)* • 1 mark
	3 marks	**1 mark**

		Simple and complex routine		
Questions B2a) and B2b)		**KAP**	1, 3	**CAJ**	1, 4

Model Response	KAP Marking Allocation	CAJ Marking Allocation
QUESTION B2a) Section A $= f(x-7)$	**QUESTION B2a)** [recall, access, selection of mathematical definitions, rules and procedures] $the\ correct\ transformation\ "-7"\ is\ identified$ • 1 mark	**QUESTION B2a)** [appropriate interpretation and use of mathematical terminology, symbols and conventions] $function\ notation\ has\ been\ used\ correctly\ in\ \ part\ a),$ $i.e.\ "x \pm a"\ \ in\ brackets\ where\ a\ may\ be\ any\ value$ $and\ the\ function\ symbol\ f\ \ outside\ of\ the\ brackets$ $i.e.\ "f\ (x \pm a)"$ • 1 mark
QUESTION B2b) Section B transformations: $horizontal\ translation, 7\ units\ right$ $f(x-7)$ $reflection\ in\ x\ axis$ $-f(x)$ $vertical\ translation, 9\ units\ up$ $f(x)+9$ Section B description: $-f(x-7)+9$ Note: allow for follow-through errors when marking part b).	**QUESTION B2b)** [numerical calculations, spatial sense and algebraic facility] $all\ 3\ transformations\ are\ correctly\ identified,$ $i.e.\ "f(x\ -\ 7)"\ \ and\ "-f(x)"\ and\ "f(x)+9"$ • 2 marks Note: one mark may be awarded if only 1 or 2 of the transformations are correctly identified in some manner.	**QUESTION B2b)** [analysis and translation of information from one representation to another] $any\ identified\ transformations\ are\ combined\ into\ a$ $single\ expression\ using\ correct\ function\ notation$ $(KAP\ errors\ may\ be\ present)$ • 1 mark
	3 marks	**2 marks**

	Simple routine			
Questions B3a) and B3b)	**KAP**	1, 2, 3	**CAJ**	1, 4

Model Response	KAP Marking Allocation	CAJ Marking Allocation
QUESTION B3a) domain $[20, 60]$ or $(20, 60)$ or $20 \leq x \leq 60$ or $x \geq 20$ *and* $x \leq 60$ or $20 < x < 60$ or $x > 20$ *and* $x < 60$ range $[0, 40]$ or $(0, 40)$ or $0 \leq y \leq 40$ or $y \geq 0$ *and* $y \leq 40$ or $0 < y < 40$ or $y > 0$ *and* $y < 40$ Note: a mixture of inclusive or exclusive inequality symbols is permitted due to visual interpretation of the graphic	**QUESTION B3a)** [application of mathematical definitions, rules and procedures] *both domain values* "20 and 60" *are identified, the notation used is not an issue here* • 1 mark *both range values* "0 and 40" *are identified, the notation used is not an issue here* • 1 mark	**QUESTION B3a)** [appropriate interpretation and use of mathematical terminology, symbols and conventions] *correct interval notation symbols have been used in either the domain or the range (the notation must define an enclosed region)* • 1 mark

QUESTION B3b) gradient: $m = \frac{y_2 - y_1}{x_2 - x_1} \left(= \frac{rise}{run}\right)$ $= \frac{80 - 20}{220 - 40}$ $= \frac{60}{180}$ $= \frac{1}{3}$ $\therefore y = \frac{1}{3}x + c$ substitute (40, 20): $20 = \frac{1}{3} \times 40 + c$ $20 = \frac{40}{3} + c$ $60 = 40 + 3c$ $20 = 3c$ $c = \frac{20}{3}$ Answer: $y = \frac{1}{3}x + \frac{20}{3}$ Note: allow for follow-through errors when marking part b).	**QUESTION B3b)** [recall, access, selection of mathematical definitions, rules and procedures] [application of mathematical definitions, rules and procedures] [numerical calculations, spatial sense and algebraic facility] *the correct gradient* " $\frac{1}{3}$ " *or equivalent fraction or decimal is stated* • 1 mark *substitution of either coordinates* $(40, 20)$ *or* $(220, 80)$ *is evident* • 1 mark *the correct y intercept* " $\frac{20}{3}$ " *or equivalent fraction or decimal is stated* • 1 mark *the correct line equation is stated in the form* $y = mx + c$ • 1 mark **6 marks**	**QUESTION B3b)** [appropriate interpretation and use of mathematical terminology, symbols and conventions] [use of mathematical reasoning to develop coherent and logical sequences within a response] *either the gradient or the y intercept fraction has been expressed in the simplest form or decimal values are correctly rounded (KAP errors may be present)* • 1 mark *reasoning is given to support the line equation ie how the line equation was obtained (KAP errors may be present)* • 1 mark **3 marks**

	Simple routine and complex routine			
Questions B4a) and B4b) and B4c) and B4d)	**KAP**	1, 2, 3	**CAJ**	4

Model Response	KAP Marking Allocation	CAJ Marking Allocation
QUESTION B4a) $\sqrt{3}$	**QUESTION B4a)** [use of stated rules and procedures] *the correct answer "* $\sqrt{3}$ *" is stated* • 1 mark	
QUESTION B4b) $\frac{7\pi}{20} \times \frac{180}{\pi} = 7 \times 9$ $= 63°$	**QUESTION B4b)** [recall, access, selection of mathematical definitions, rules and procedures] [numerical calculations, spatial sense, algebraic facility] *the correct rule "* $\times \frac{180}{\pi}$ *" has been stated* *or a correct procedure is explained* • 1 mark *the correct answer "*$63°$*" is stated* • 1 mark	**QUESTION B4b)** [use of mathematical reasoning to develop sequences within a response] *reasoning is given to support the angle value in degrees* *i.e. how the angle was obtained* *(KAP errors may be present)* • 1 mark
QUESTION B4c) $\sin\theta = \frac{opposite}{hypotenuse}$ $\sin 60° = \frac{h}{10}$ $h = 10 \times \sin 60°$ $h = 10 \times \frac{\sqrt{3}}{2}$ $h = 5 \times \sqrt{3}$ or $h = 5\sqrt{3}\ cm$ Note: allow for follow-through errors when marking part c).	**QUESTION B4c)** [recall, access, selection of mathematical definitions, rules and procedures] [application of mathematical definitions, rules and procedures] [numerical calculations, spatial sense and algebraic facility] *the sine ratio has been identified as appropriate* • 1 mark *the exact value of the used trigonometric ratio has been stated* • 1 mark *the correct simplified h value is stated* • 1 mark	**QUESTION B4c)** [use of mathematical reasoning to develop coherent and logical sequences within a response] *reasoning is given to support the value of h obtained* *i.e. how the value of h was obtained* *(KAP errors may be present)* • 1 mark

QUESTION B4d) $\sqrt{3}\tan x + 1 = 0$ $\sqrt{3}\tan x = -1$ $\tan x = -\frac{1}{\sqrt{3}}$ the basic angle is $\frac{\pi}{6}$ $(or\ 30°)$ tan is negative and $0 \leq x \leq \pi$ - this requires the angle to be in quadrant 2 $x = \pi - \frac{\pi}{6}$ $(180° - 30°)$ $x = \frac{6\pi}{6} - \frac{\pi}{6}$ $(150°)$ $x = \frac{5\pi}{6}$ Notes: • allow for follow-through errors when marking part d) • a final answer in degrees is not acceptable.	**QUESTION B4d)** [recall, access, selection of mathematical definitions, rules and procedures] [application of mathematical definitions, rules and procedures] [numerical calculations, spatial sense and algebraic facility] *the equation has been rearranged correctly to make* $\tan x$ *the subject* • 1 mark *the basic acute angle has been identified* • 1 mark *the domain is considered and quadrant* 2 *has been identified* • 1 mark *a correct simplified angle in quadrant* 2 *is stated* • 1 mark **10 marks**	**QUESTION B4d)** [use of mathematical reasoning to develop coherent, concise and logical sequences within a response] *evidence of the steps used in the equation rearrangement is provided (KAP errors may be present)* • 1 mark *reasoning is given to support the solution obtained i.e. how the solution was obtained (KAP errors may be present)* • 1 mark **4 marks**

	Simple routine			
Question B5	**MAP**	1	**CAJ**	4, 5

Model Response	MAP Marking Allocation	CAJ Marking Allocation
QUESTION B5 IQR of both groups is the same $\therefore 13 - y = 15 - 10$ $13 - y = 5$ $-y = -8$ $y = 8$ The range differs by 2 The range of Group 2 $= 18 - 0$ $= 18$ $\therefore$ the range of Group 1 $= 16$ or 20 (differs by 2) 20 is not possible (too large producing $x < 0$) $\therefore$ the range of Group 1 $= 16$ $19 - x = 16$ $-x = -3$ $x = 3$	**QUESTION B5** [use of problem-solving strategies to interpret, clarify and develop responses] *the correct value of y is stated* "8" • 2 marks OR *a problem solving strategy is evident with error in the solution of y* • 1 mark *the Group 1 range is stated as "16" and/or "20" or equivalent identification* • 1 mark *correct value of x is stated* "3" • 1 mark	**QUESTION B5** [use of mathematical reasoning to develop sequences within a response] [justification of procedures, decisions or results] *a reason is given that explains how the range of Group 1 is obtained* • 1 mark *justification is given to support either value x or y, i.e. how a value was obtained is provided (MAP errors may be present)* • 1 mark
	4 marks	**2 marks**

	Simple routine			
Question B6	MAP	1	**CAJ**	4

Model Response	MAP Marking Allocation	CAJ Marking Allocation
QUESTION B6 To find a: substitute $(-1, 0)$ $0 = a(2)^2 + 8$ $0 = 4a + 8$ $4a = -8$ $a = -2$ $\therefore \quad y = -2(x + 3)^2 + 8$ To find b: (the y-intercept) substitute $x = 0$ $y = -2(3)^2 + 8$ $y = -18 + 8$ $y = -10$ $\therefore \quad b = -10$ To find c: $y = a(x + 3)^2 + 8$ compare this with $y = a(x - h)^2 + k$ $\therefore (h, k) = (-3, 8) \quad (the\ turning\ point)$ by symmetry, $c = -5$ An algebraic method to find c is also acceptable. Note: allow for follow-through errors when marking.	**QUESTION B6** [use of problem-solving strategies to interpret, clarify and analyse problems to develop responses] $substitution\ of\ the\ coordinates\ (-1,0)\ into\ the\ function\ is\ evident$ • 1 mark $the\ correct\ value\ of\ a\ is\ stated\ "-2"$ • 1 mark $substitution\ of\ x = 0\ into\ the\ function\ is\ evident$ • 1 mark $the\ correct\ value\ of\ b\ is\ stated$ "−10" • 1 mark $the\ correct\ value\ of\ c\ is\ stated$ "−5" • 1 mark **5 marks**	**QUESTION B6** [use of mathematical reasoning to develop coherent, concise and logical sequences within a response] $reasoning\ is\ given\ to\ support\ each\ value\ of\ a, b\ and\ c,\ i.e.\ how\ each\ value\ was\ obtained\ is\ provided\ (MAP\ errors\ may\ be\ present)$ • 1 mark for a • 1 mark for b • 1 mark for c **3 marks**

			Complex routine		
Question B7a)		**KAP**	1, 2, 3	**CAJ**	1

Model Response	**KAP Marking Allocation**	**CAJ Marking Allocation**
QUESTION B7a) $g\left(f(x)\right) = g\,(x+3)$ $= (x+3)^2 - 4$ $= x^2 + 6x + 9 - 4$ $= x^2 + 6x + 5$ Notes: • if "$x^2 + 6x + 5$" is stated as the final answer then the expansion is assumed to have been done and the mark awarded • allow for follow-through errors in the simplification if the expansion of the squared bracket step is incorrect.	**QUESTION B7a)** [recall, access, selection of mathematical definitions, rules and procedures] [application of mathematical definitions, rules and procedures] [numerical calculations, spatial sense and algebraic facility] *correct substitution of* $(x+3)$ *into* $g(x)$ • 1 mark *correct algebraic expansion of squared bracket* "$x^2 + 6x + 9$ " • 1 mark *correct algebraic simplification of final expression* "$x^2 + 6x + 5$" • 1 mark	**QUESTION B7a)** [appropriate interpretation and use of mathematical terminology, symbols and conventions] *use of composite function notation is evident in part a)* • 1 mark
	3 marks	**1 mark**

		Complex non-routine		
Question B7b)		**MAP**	1	**CAJ**	4

Model Response	MAP Marking Allocation	CAJ Marking Allocation
QUESTION B7b) $f\left(g(x)\right) = f\left(x^2 - 4\right)$ $= x^2 - 4 + 3$ $= x^2 - 1$ $f\left(g(x)\right) = g\left(f(x)\right)$ $\therefore x^2 - 1 = x^2 + 6x + 5$ $-1 = 6x + 5$ $-6 = 6x$ $x = -1$ Note: allow for follow-through errors when marking part b).	**QUESTION B7b)** [use of problem-solving strategies to interpret, clarify and analyse problems to develop responses] *correct expression for* $f\left(g(x)\right)$ *is stated* "$x^2 - 1$" or "$x^2 - 4 + 3$" • 1 mark $f\left(g(x)\right)$ *is equated with* $g\left(f(x)\right)$ • 1 mark *correct answer for* x *is stated* "-1" • 1 mark **3 marks**	**QUESTION B7b)** [use of mathematical reasoning to develop coherent, concise and logical sequences within a response] *reasoning is given to support the value of* x*,* *i.e. how the value of* x *was obtained is provided* *(MAP errors may be present)* • 1 mark **1 mark**

	Complex routine			
Question B8	**MAP**	1	**CAJ**	3, 4

Model Response	MAP Marking Allocation	CAJ Marking Allocation
QUESTION B8 Let $a = the\ number\ of\ 13\ passenger\ buses$ $b = the\ number\ of\ 24\ passenger\ buses$ $a + b = 25 \quad (1)$ $13a + 24b = 479 \quad (2)$ rearranging (1): $a = 25 - b$ substitute into (2): $13(25 - b) + 24b = 479$ $325 - 13b + 24b = 479$ $325 + 11b = 479$ $11b = 154$ $b = \frac{154}{11}$ $b = 14$ substitute into (1): $a + 14 = 25$ $a = 11$ Answer: there are 11 of the 13-passenger buses and there are 14 of the 24-passenger buses Note: allow for follow-through errors when marking.	**QUESTION B8** [use of problem-solving strategies to interpret, clarify and analyse problems to develop responses] *two correct algebraic equations have been stated that describe the total number of minibuses and the total number of passengers, (the equations must involve the use of variables)* • 1 mark *an algebraic strategy has been used to determine an answer (substitution or elimination)* • 1 mark *the correct answer for 24 passenger buses is obtained* "14" • 1 mark *the correct answer for 13 passenger buses is obtained* "11" • 1 mark **4 marks**	**QUESTION B8** [analysis and translation of information from one representation to another] [appropriate interpretation and use of mathematical terminology, symbols and conventions] [use of mathematical reasoning to develop coherent, concise and logical sequences within a response] *identification of variables is present i.e. any introduced variables are explained ("the number of " does not need to be included)* • 1 mark *algebraic reasoning is given to support at least one of the answer values i.e. how at least one of the answer values was obtained is provided (MAP errors may be present)* • 1 mark **2 marks**

			Simple non-routine	
Question D1	**KAP**	2, 3, 4	**CAJ**	3, 4

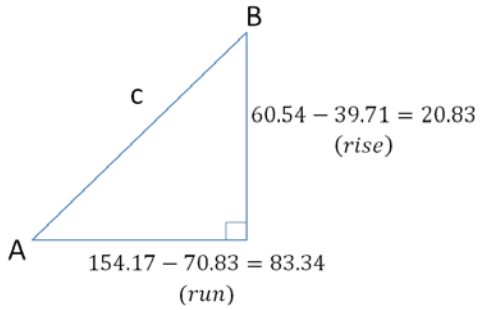
Model Response	KAP Marking Allocation	CAJ Marking Allocation
QUESTION D1 there are five equally spaced arms, so the angle between each arm $= \frac{360°}{5}$ $= 72°$ the other two angles in the isosceles triangle formed between any two arms (these angles are equal): $= \frac{(180-72)°}{2}$ $= \frac{(108)°}{2}$ $= 54°$ using the sine rule: $\frac{a}{\sin 54} = \frac{10}{\sin 72}$ $a = \frac{10}{\sin 72} \times \sin 54$ $= 8.51\ cm$ Notes: • there is to be no penalty if radian measure is used instead of degrees for the angles • allow for follow-through errors when marking.	**QUESTION D1** [application of mathematical definitions, rules and procedures] [numerical calculations, spatial sense and algebraic facility] [appropriate selection and use of technology] "72°" *has been stated as one of the required angles* (1.2566 radians) • 1 mark "54°" *has been stated as another required angle* (0.94248 radians) • 1 mark *the sine rule has been used correctly to obtain the length of one arm, a* (8.5–8.51 are acceptable values) • 1 mark **3 marks**	**QUESTION D1** [analysis and translation of information from one representation to another] [use of mathematical reasoning to develop coherent and logical sequences within a response] *the information has been analysed to identify two different feasible angles* $(both\ are\ < 90°)$ $(KAP\ errors\ may\ be\ present)$ • 1 mark *angles and lengths are substituted appropriately into the sine rule and reasoning is shown to support the value of a i.e. how a was obtained is stated* • 1 mark **2 marks**

	Simple routine and simple non-routine			
Questions D2a) D2b) and D2c)	**KAP**	2, 3, 4	**CAJ**	2, 3, 4

Model Response	KAP Marking Allocation	CAJ Marking Allocation
QUESTION D2a) the number of students in the class: $1 + 2 + 3 + 4 + 5 + 1 + 3 + 2$ $= 21\ students$	**QUESTION D2a)** [numerical calculations, spatial sense and algebraic facility] $the\ value\ "21"\ is\ stated$ $(the\ unit\ "students"\ is\ not\ required)$ • 1 mark	**QUESTION D2a)** [use of mathematical reasoning] $how\ the\ answer\ was\ obtained\ is\ evident$ • 1 mark
QUESTION D2b) the five number summary values are evident either in the boxplot drawn or written separately $min = 8$ $Q_1 = 10$ $median = 12$ $Q_3 = 13.5$ $max = 15$ Note: allow for follow-through errors when marking CAJ.	**QUESTION D2b)** [application of mathematical definitions, rules and procedures] [appropriate selection and accurate use of technology] $all\ values\ in\ the\ five\ number\ summary\ are\ correct$ • 3 marks $OR\ 3\ or\ 4\ of\ the\ values\ are\ correct$ • 2 marks $OR\ 1\ or\ 2\ of\ the\ values\ are\ correct$ • 1 mark	**QUESTION D2b)** [organisation and presentation of information in a variety of representations] $the\ shape\ of\ the\ drawing\ is\ recognisable\ as\ a\ boxplot\ with\ whiskers\ and\ box\ section$ • 1 mark $the\ five\ number\ summary\ values\ identified\ are\ correctly\ located\ on\ the\ scale$ • 1 mark

QUESTION D2c) $\bar{x} = 11.67$ $\sigma = 1.94$ $\bar{x} + \sigma = 11.67 + 1.94$ $= 13.61$ Since no part marks were given, students more than one standard deviation above the mean obtained 14 marks or 15 marks. This is 14 marks → 3 students 15 marks → 2 students a total of 5 students. • if the sample standard deviation (1.98) is used, no penalty in awarding marks • if the sample standard deviation is used, the model response calculation becomes $\bar{x} = 11.67$ $\sigma = 1.98$ $\bar{x} + \sigma = 11.67 + 1.98$ $= 13.65$ The subsequent reasoning and final answer of "5 students" remains the same. Note: allow for follow-through errors when marking part c).	**QUESTION D2c)** [application of mathematical definitions, rules and procedures] [numerical calculations, spatial sense and algebraic facility] [appropriate selection and accurate use of technology] *either the mean value, "11.67", or the standard deviation value, "1.94", are stated* • 1 mark (11.6–11.7 are acceptable mean values) (1.9–2 are acceptable standard deviation values) *the mean and standard deviation are added together to obtain the sum value, i.e. the sum is stated* • 1 mark *the correct number of students, "5", is stated as the final answer* • 1 mark **7 marks**	**QUESTION D2c)** [use of mathematical reasoning to develop coherent, concise and logical sequences within a response] [coherent, concise and logical justification of procedures, decisions and results] *reasoning is given to support the sum value that is one standard deviation above the mean i.e. how the value was obtained is provided (KAP errors may be present)* • 1 mark *reasoning is given to support the final answer (KAP errors may be present)* • 1 mark **5 marks**

	Simple routine			
Question D3	**MAP**	1	**CAJ**	3, 4

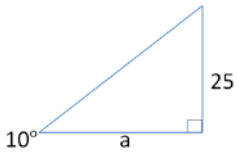
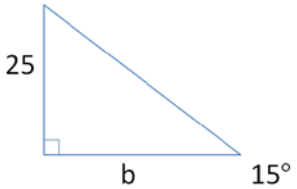
Model Response	MAP Marking Allocation	CAJ Marking Allocation
QUESTION D3 Use a graphics calculator to find the points of intersection between the line: $y = \frac{1}{4}x + 22$ and the parabola: $y = -\frac{1}{60}x^2 + 4x - 160$ → A$(70.83, 39.71)$ and B$(154.17, 60.54)$ To many decimal places the coordinates are: A(70.831667, 39.707917) B(154.16833, 60.542083) Use Pythagoras' theorem (or Distance formula):  $c^2 = 20.83^2 + 83.34^2$ $c = \sqrt{7379.44}$ $c = 85.90\ c$ Note: allow for follow-through errors when marking.	**QUESTION D3** [use of problem-solving strategies to interpret, clarify and analyse problems to develop responses] *the points of intersection are correctly stated* • 1 mark for A • 1 mark for B (a range of values are acceptable for the coordinates) A (70–71, 39–40) B (154–155, 60–61) *the correct AB value, based on the points of intersection used, is stated* • 2 marks (85.89–85.91 are acceptable AB values) **4 marks**	**QUESTION D3** [analysis and translation of information from one representation to another] [use of mathematical reasoning to develop coherent and logical sequences within a response] *the points of intersection obtained are correctly identified as A and B from the diagram* • 1 mark *identification of either two lengths of a right-triangle (rise and run), OR two distances ($y_2 - y_1$ and $x_2 - x_1$)* • 1 mark for "*rise*" stated or "$y_2 - y_1$" • 1 mark for "*run*" stated or "$x_2 - x_1$" *substitution of values into Pythagoras' theorem or the Distance formula is shown (MAP errors may be present)* • 1 mark **4 marks**

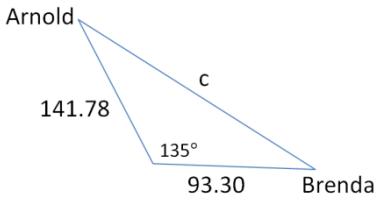
	Simple routine			
Question D4	**MAP**	1	**CAJ**	1, 3

Model Response	MAP Marking Allocation	CAJ Marking Allocation
QUESTION D4 Use a graphics calculator to graph the function: $S = 3\cos\left(\frac{2\pi t}{365}\right) + 25$ setting the viewing window as: *t* values between 0 and 365 (days in a year) *S* values between 10 and 40 (temp of a tropical island) Use the graph solve feature to set *S* = 23 and find the associated *t* value: *S* = 23 → *t* = 133.64 Answer: The water temperature will first become 23°C after 133.64 days 134 days — the first actual day	**QUESTION D4** [use of problem-solving strategies to interpret, clarify and analyse problems to develop responses] *a suitable problem solving strategy is used, either –* *– use a graphics calculator* *– algebraic method* *– guess and check method* *– state both the view windows required in a calculator such as* $[0, 365]$ *and* $[10, 40]$ *Demonstrating the use of a strategy* • 1 mark	**QUESTION D4** [analysis and translation of information from one representation to another] [appropriate interpretation and use of mathematical terminology] *it is apparent that* "23" *has been substituted into the S value of the equation* • 1 mark

OR Solve the equation algebraically: $S = 3\cos\left(\frac{2\pi t}{365}\right) + 25$ substitute S = 23: $23 = 3\cos\left(\frac{2\pi t}{365}\right) + 25$ $-2 = 3\cos\left(\frac{2\pi t}{365}\right)$ $-\frac{2}{3} = \cos\left(\frac{2\pi t}{365}\right)$ $\frac{2\pi t}{365} = cos^{-1}\left(-\frac{2}{3}\right)$ $\frac{2\pi t}{365} = 2.3005$ $2\pi t = 839.6913$ $t = 133.64$	*the correct final answer is stated* (Any rounded version of the day is to be awarded the mark). • 1 mark (133–134 are acceptable day values)	*a written account to support the number of days is evident, for example –* *a sketch or graph with location is shown* *OR* *algebraic manipulation that includes the correct use of inverse cosine* • 1 mark
	2 marks	**2 marks**

	Complex routine			
Question D5	**MAP**	1	**CAJ**	3, 4

Model Response	MAP Marking Allocation	CAJ Marking Allocation
QUESTION D5 Arnold:  $\tan 10° = \frac{25}{a}$ $a = \frac{25}{\tan 10°}$ $a = 141.78\ m$ Brenda:  $\tan 15° = \frac{25}{b}$ $b = \frac{25}{\tan 15°}$ $b = 93.30\ m$ Using the Cosine Rule:	**QUESTION D5** [use of problem-solving strategies to interpret, clarify and analyse problems to develop responses] *correctly found Arnold's distance from tower* • 1 mark (141.7–141.8 are acceptable Arnold values) *correctly found Brenda's distance from tower* • 1 mark (93.3–93.4 are acceptable Brenda values)	**QUESTION D5** [analysis and translation of information from one representation to another] [use of mathematical reasoning to develop coherent, concise and logical sequences within a response] *use of trigonometry to determine either Arnold's or Brenda's distance from the tower is evident* • 1 mark

 $c^2 = 141.78^2 + 93.30^2 - 2\times141.78\times93.30\times\cos 135°$ $= 47513.78$ $c = \sqrt{47513.78}$ $= 217.98\ m$ Note: allow for follow-through errors when marking.		*correctly found the required angle between Arnold and Brenda, "135°" is stated* • 1 mark *the correct distance between Arnold and Brenda is stated* • 1 mark (217.9–218.1 are acceptable values)		*substitution of values into the cosine rule is evident (MAP errors may be present)* • 1 mark
		4 marks		**2 marks**

	Simple routine and complex non-routine			
Question D6	**MAP**	1, 3	**CAJ**	1, 3, 5, 6

Model Response	MAP Marking Allocation	CAJ Marking Allocation
QUESTION D6a) The equation required may be obtained by using a graphics calculator and the two pairs of data values (points) supplied in the table. Using a graphics calculator the values can be modelled using linear regression $y = mx + c$ $\therefore y = -12.5\,x + 8255$ Or Algebraically: $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{3505 - 5880}{380 - 190}$ $m = -\frac{2375}{190}$ $m = -12.5$ Use $y = mx + c$ $y = -12.5\,x + c$ substitute $(190, 5880)$: $5880 = -12.5 \times 190 + c$ $5880 = -2375 + c$ $c = 5880 + 2375$ $c = 8255$ $\therefore y = -12.5\,x + 8255$ Note: allow for follow-through errors when marking any algebraic response.	**QUESTION D6a)** [use of data to synthesise mathematical models] *the gradient, "−12.5" has been stated* • 1 mark *the y − intercept, "8255" has been stated* • 1 mark	**QUESTION D6a)** [appropriate interpretation and use of mathematical terminology, symbols and conventions] [analysis and translation of information from one representation to another] *the use of a graphics calculator has been stated or an algebraic response is evident* *(MAP errors may be present)* • 1 mark *the linear model with calculated parameters (m and c) is appropriately stated using the variables x and y* *(MAP errors may be present)* • 1 mark

QUESTION D6b) Using a graphics calculator: $x = 170 \rightarrow y = 6130$ $\therefore p = 6130$ $y = 4505 \rightarrow x = 300$ $\therefore q = 300$ Or algebraically: To find p: let $x = 170$ $y = -12.5 \times 170 + 8255$ $y = 6130$ $\therefore p = 6130$ To find q: let $y = 4505$ $4505 = -12.5\,x + 8255$ $-3750 = -12.5\,x$ $x = \frac{-3750}{-12.5}$ $= 300$ $\therefore q = 300$ Notes: • allow for follow-through errors when marking part b) • when marking CAJ for this item, Section D instructions state "Questions worth more than one mark require working to be shown to support answers".	**QUESTION D6b)** [use of data to synthesise mathematical models] *the correct value of p has been stated* • 1 mark *the correct value of q has been stated* • 1 mark	**QUESTION D6b)** [coherent and logical justification of procedures, decisions and results] *use of the value* 170 *is shown to produce the p value i.e. how the p value was obtained is stated* • 1 mark *use of the value* 4505 *is shown to produce the q value i.e. how the q value was obtained is stated* • 1 mark

QUESTION D6c) Use $y = -12.5x + 8255$ Cost of burning 250 ha: let $x = 250$ $y = -12.5 \times 250 + 8255$ $= 5130$ i.e. cost/ha = \$5130/ha $\therefore$ maintenance cost $= 5130 \times 250$ $= \$\ 1282\ 500$ Cost of burning 200 ha: let $x = 200$ $y = -12.5 \times 200 + 8255$ $= 5755$ i.e. cost/ha = \$5755/ha $\therefore$ maintenance cost $= 5755 \times 200$ $= \$\ 1151\ 000$ The saving in maintenance cost $= \$1\ 282\ 500 - \$1\ 151\ 000$ $= \$131\ 500$ Answer: YES, the suggestion made would save at least \$100 000. Note: allow for follow-through errors when marking part c).	**QUESTION D6c)** [use of problem-solving strategies to interpret, clarify and analyse problems to develop responses] "\$5130" *has been stated for* 250 *ha* • 1 mark *the maintenance cost has been stated for* 250 *ha* "\$ 1282 500" • 1 mark "\$5755" *has been stated for* 200 *ha* • 1 mark *the maintenance cost has been stated for* 200 *ha* "\$ 1151 000" • 1 mark *the overall maintenance saving,* "\$131 500" *has been stated* • 1 mark **9 marks**	**QUESTION D6c)** [coherent, concise and logical justification of the procedures, decisions and results] *a coherent and logical procedure is evident in the response* *(MAP errors may be present)* • 1 mark *a decision is stated that is supported by logical justification* *(MAP errors may be present)* • 1 mark **6 marks**

Appendix 3: Assessment glossary

Item	Description
calculate	determine or find (e.g. a number, answer) by using mathematical processes; ascertain/determine from given facts, figures or information
create	bring something into being or existence; produce or evolve from one's own thought or imagination; reorganise or put elements together into a new pattern or structure or to form a coherent or functional whole
convert	to change into something of different form or properties; transmute; transform
data	In the context of the Mathematics B syllabus, data are thought to include documented information or evidence of any kind that lends itself to mathematical interpretation; data may be quantitative or qualitative
design	produce, e.g. a plan, simulation, model, project; plan; plan or fashion; form or conceive in the mind
determine	establish, conclude or ascertain after consideration, observation, investigation or calculation; obtain the only possible answer; decide or come to a resolution
develop	elaborate, expand or enlarge in detail; add detail and fullness to; cause to become more complex or intricate
identify	distinguish; locate, recognise and name; establish or indicate what something is; provide an answer from a number of possibilities; recognise and state a distinguishing factor or feature
justify	give reasons or evidence to support an answer, response or conclusion; show or prove how an argument, statement or conclusion is right or reasonable
predict	give an expected result of an upcoming action or event; suggest what may happen based on available information
show	provide the relevant reasoning to support a response
sketch	a simple or preliminary drawing, diagram or graph, that provides the essential features without all the details
solve	obtain the answer/s using algebraic and/or numerical and/or graphical methods
state	give a specific name, value, or other brief answer without explanation or calculation
structure	give a pattern, organisation or arrangement to; construct or arrange according to a plan