
Formula book

Specialist Mathematics 2025

Mensuration			
circumference of a circle	$C = 2\pi r$	area of a circle	$A = \pi r^2$
area of a parallelogram	$A = bh$	area of a trapezium	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	total surface area of a cone	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	surface area of a sphere	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	volume of a cylinder	$V = \pi r^2 h$
volume of a prism	$V = Ah$	volume of a pyramid	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Graph equations		
quadratic	$y = a(x-h)^2 + k$	$y = a(x-x_1)(x-x_2)$
cubic	$y = a(x-h)^3 + k$	$y = a(x-x_1)(x-x_2)(x-x_3)$
circle	$(x-h)^2 + (y-k)^2 = r^2$	
square root	$y = a\sqrt{x-h} + k$	
reciprocal	$y = \frac{a}{(x-h)} + k$	
exponential	$y = r^{(x-h)} + k$ (where $r > 0$)	
logarithmic	$y = \log_a(x-h) + k$ (where $a > 1$)	
trigonometric	$y = a \sin(b(x-h)) + k$	$y = a \cos(b(x-h)) + k$

Logarithms		
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$	
logarithmic laws and definitions	$\log_a(x) + \log_a(y) = \log_a(xy)$	$\log_a(x^n) = n \log_a(x)$
	$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$	$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$
	$\log_a(1) = 0$	$\log_a(a) = 1$

Calculus		
$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$		$\int_a^b f(x) dx \approx \text{limit of sums } \sum_i f(x_i) \delta x_i$ $\int_a^b f(x) dx = F(b) - F(a)$
$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$		$\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$ $\int k f(x) dx = k \int f(x) dx$
$\frac{d}{dx} x^n = nx^{n-1}$		$\int x^n dx = \frac{x^{n+1}}{n+1} + c$ for $n \neq -1$
$\frac{d}{dx} e^x = e^x$	$\frac{d}{dx} e^{f(x)} = f'(x) e^{f(x)}$	$\int e^x dx = e^x + c$
$\frac{d}{dx} \ln(x) = \frac{1}{x}$	$\frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)}$	$\int \frac{1}{x} dx = \ln(x) + c$ for $x > 0$
$\frac{d}{dx} \sin(x) = \cos(x)$	$\frac{d}{dx} \sin(f(x)) = f'(x) \cos(f(x))$	$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx} \cos(x) = -\sin(x)$	$\frac{d}{dx} \cos(f(x)) = -f'(x) \sin(f(x))$	$\int \cos(x) dx = \sin(x) + c$
chain rule	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	
quotient rule	$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$	
trapezoidal rule	$\int_a^b f(x) dx \approx \frac{w}{2} [f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) + \dots + f(x_{n-1})) + f(x_n)]$ where $w = \frac{b-a}{n}$	

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{1}{2} bc \sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Statistics		
binomial theorem	$(x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \dots + \binom{n}{r}x^{n-r}y^r + \dots + y^n$	
binomial probability	$P(X = r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable X	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
	standard deviation	$\sqrt{Var(X)}$
continuous random variable X	mean	$E(X) = \mu = \int_{-\infty}^{\infty} x p(x) dx$
	variance	$Var(X) = \sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 p(x) dx$
Bernoulli distribution	mean	p
	variance	$p(1-p)$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for p	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$	
approximate margin of error	$z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	
complementary probability	$P(\bar{A}) = 1 - P(A)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A)P(B)$	
conditional probability	$P(A \cap B) = P(A B)P(B)$	

Additional Calculus for Specialist Mathematics

$\frac{d}{dx} \ln(x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln x + c$ for $x \neq 0$		
$\frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)}$	$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$ for $f(x) \neq 0$		
$\frac{d}{dx} \tan(x) = \sec^2(x)$	$\int \sec^2(x) dx = \tan(x) + c$		
$\frac{d}{dx} \sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$		
$\frac{d}{dx} \cos^{-1}\left(\frac{x}{a}\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c$		
$\frac{d}{dx} \tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$		
integration by parts	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$		
volume of a solid of revolution	about the x-axis $V = \pi \int_a^b [f(x)]^2 dx$		
	about the y-axis $V = \pi \int_a^b [f(y)]^2 dy$		
Simpson's rule	$\int_a^b f(x) dx \approx \frac{w}{3} [f(x_0) + 4[f(x_1) + f(x_3) + \dots] + 2[f(x_2) + f(x_4) + \dots] + f(x_n)]$ <p>where $w = \frac{b-a}{n}$</p>		
simple harmonic motion	If $\frac{d^2x}{dt^2} = -\omega^2 x$ then $x = A \sin(\omega t + \alpha)$ or $x = A \cos(\omega t + \beta)$		
	<table style="width: 100%; border: none;"> <tr> <td style="border: none;">$v^2 = \omega^2 (A^2 - x^2)$</td> <td style="border: none; text-align: center;">$T = \frac{2\pi}{\omega}$</td> <td style="border: none; text-align: right;">$f = \frac{1}{T}$</td> </tr> </table>	$v^2 = \omega^2 (A^2 - x^2)$	$T = \frac{2\pi}{\omega}$
$v^2 = \omega^2 (A^2 - x^2)$	$T = \frac{2\pi}{\omega}$	$f = \frac{1}{T}$	
acceleration	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$		

Additional Trigonometry for Specialist Mathematics

Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \operatorname{cosec}^2(A)$
angle sum and difference identities	$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 1 - 2\sin^2(A)$ $= 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}(\cos(A - B) - \cos(A + B))$ $\cos(A)\cos(B) = \frac{1}{2}(\cos(A - B) + \cos(A + B))$ $\sin(A)\cos(B) = \frac{1}{2}(\sin(A + B) + \sin(A - B))$ $\cos(A)\sin(B) = \frac{1}{2}(\sin(A + B) - \sin(A - B))$

Graph equations for Specialist Mathematics

sphere	$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$	
circle	$(x - h)^2 + (y - k)^2 = r^2$	
ellipse	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$	
hyperbola	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$	$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$

Additional Statistics for Specialist Mathematics	
inclusion-exclusion principle	$ A \cup B = A + B - A \cap B $ $ A \cup B \cup C = A + B + C - A \cap B - A \cap C - B \cap C + A \cap B \cap C $
permutation	${}^n P_r = \frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \dots \times (n-r+1)$
combination	${}^n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$
probability density function of the exponential distribution	$f(t) = \lambda e^{-\lambda t}$ for $t \geq 0, \lambda > 0$
exponential random variable	mean $\frac{1}{\lambda}$
	standard deviation $\frac{1}{\lambda}$
sample means	mean μ
	standard deviation $\frac{\sigma}{\sqrt{n}}$
approximate margin of error	$E = z \frac{s}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}} \right)$

Real and complex numbers		
complex number forms	$z = a + bi = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$	
modulus	$ z = r = \sqrt{a^2 + b^2}$	
argument	$\operatorname{Arg}(z) = \theta,$ $\tan(\theta) = \frac{b}{a}, -\pi < \theta \leq \pi, a \neq 0$	$\arg(z) = \operatorname{Arg}(z) + 2\pi n, n \in \mathbb{Z}$
product	$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	
quotient	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$	
De Moivre's theorem	$z^n = r^n \operatorname{cis}(n\theta)$	

Matrices		
commutative law for addition	$A + B = B + A$	
additive identity	$A + \mathbf{0} = A$	
additive inverse	$A + (-A) = \mathbf{0}$	
multiplicative identity	$A I = A = I A$	
multiplicative inverse	$A A^{-1} = I = A^{-1} A$	
left distributive law	$A(B + C) = AB + AC$	
right distributive law	$(B + C)A = BA + CA$	
determinant	If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(A) = ad - bc$	
multiplicative inverse matrix	$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \det(A) \neq 0$	
linear transformations	dilation of factor a parallel to the x -axis and factor b parallel to the y -axis	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation of angle θ anticlockwise about the origin	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection in the line $y = x \tan(\theta)$	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

Vectors		
magnitude	$ \mathbf{a} = \left \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \right = \sqrt{a_1^2 + a_2^2}$	$ \mathbf{a} = \left \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right = \sqrt{a_1^2 + a_2^2 + a_3^2}$
direction	$\tan(\theta) = \frac{y}{x}, x \neq 0$	
unit vector	$\hat{\mathbf{n}} = \frac{\mathbf{n}}{ \mathbf{n} }$	
scalar (dot) product	$\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos(\theta)$	
	$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = a_1 b_1 + a_2 b_2$	$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$
vector equation of a line	$\mathbf{r} = \mathbf{a} + t\mathbf{d}$	
parametric equations of a line	$x = a_1 + t d_1$ $y = a_2 + t d_2$ $z = a_3 + t d_3$	
Cartesian equation of a line	$\frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$	
vector (cross) product	$\mathbf{a} \times \mathbf{b} = \mathbf{a} \mathbf{b} \sin(\theta)\hat{\mathbf{n}}$	
	$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$	
scalar projection	$\mathbf{a} \text{ on } \mathbf{b} = \mathbf{a} \cos(\theta) = \mathbf{a} \cdot \hat{\mathbf{b}}$	
vector projection	$\mathbf{a} \text{ on } \mathbf{b} = \mathbf{a} \cos(\theta)\hat{\mathbf{b}} = (\mathbf{a} \cdot \hat{\mathbf{b}})\hat{\mathbf{b}} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{b} \cdot \mathbf{b}} \right) \mathbf{b}$	
vector equation of a plane	$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	

Physical constant	
magnitude of mean acceleration due to gravity on Earth	$g = 9.8 \text{ ms}^{-2}$

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