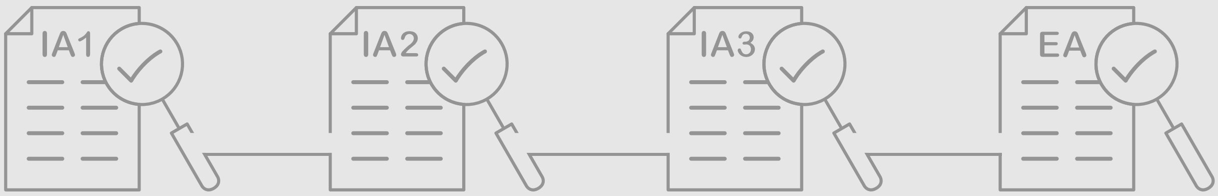
Specialist Mathematics subject report

2024 cohort

January 2025



Queensland Government

QCAA
Queensland Curriculum & Assessment Authority

For all Queensland schools

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Contents

Introduction

The annual subject reports seek to identify strengths and opportunities for improvement of internal and external assessment processes for all Queensland schools. The 2024 subject report is the culmination of the partnership between schools and the QCAA. It addresses school-based assessment design and judgments, and student responses to external assessment for General and General (Extension) subjects. In acknowledging effective practices and areas for refinement, it offers schools timely and evidence-based guidance to further develop student learning and assessment experiences for 2025.

The report also includes information about:

- how schools have applied syllabus objectives in the design and marking of internal assessments
- how syllabus objectives have been applied in the marking of external assessments
- patterns of student achievement.

The report promotes continuous improvement by:

- identifying effective practices in the design and marking of valid, accessible and reliable assessments
- recommending where and how to enhance the design and marking of valid, accessible and reliable assessment instruments
- providing examples that demonstrate best practice.

Schools are encouraged to reflect on the effective practices identified for each assessment, consider the recommendations to strengthen assessment design and explore the authentic student work samples provided.

Audience and use

This report should be read by school leaders, subject leaders, and teachers to:

- inform teaching and learning and assessment preparation
- assist in assessment design practice
- assist in making assessment decisions
- help prepare students for internal and external assessment.

The report is publicly available to promote transparency and accountability. Students, parents, community members and other education stakeholders can use it to learn about the assessment practices and outcomes for senior subjects.

Subject highlights

307
schools offered Specialist Mathematics

95.02%
agreement with provisional marks for IA1

97.96%
of students received a C or higher

Subject data summary

Subject completion

The following data includes students who completed the General subject or AS.

Note: All data is correct as at January 2025. Where percentages are provided, these are rounded to two decimal places and, therefore, may not add up to 100%.

Number of schools that offered Specialist Mathematics: 307.

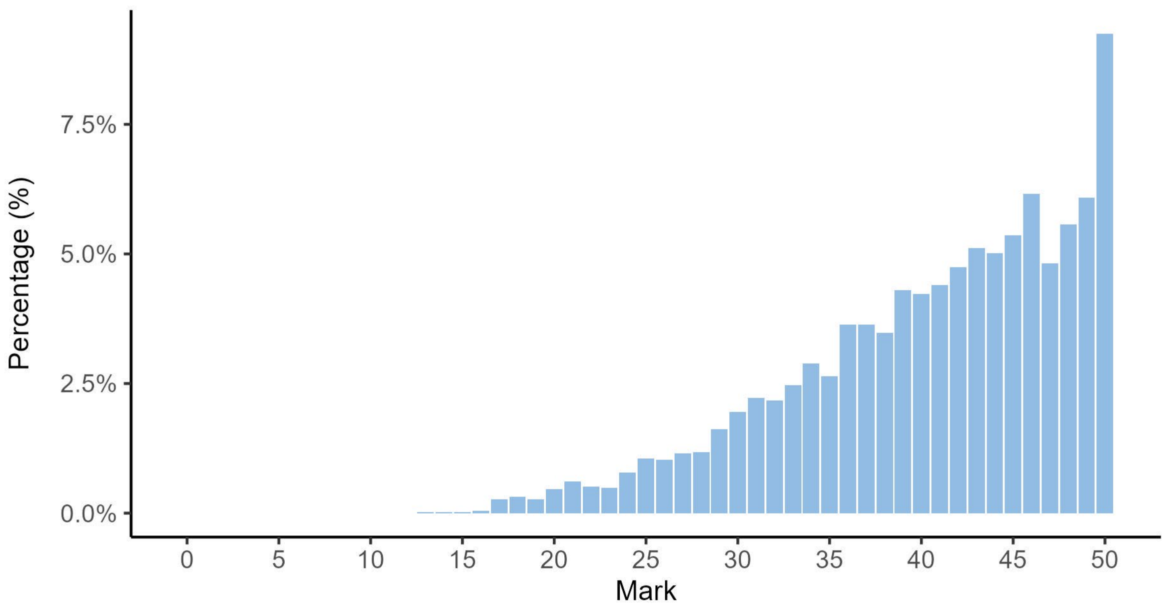
Completion of units	Unit 1	Unit 2	Units 3 and 4
Number of students completed	4,575	4,344	4,078

Units 1 and 2 results

Number of students	Satisfactory	Unsatisfactory
Unit 1	4,349	226
Unit 2	4,047	297

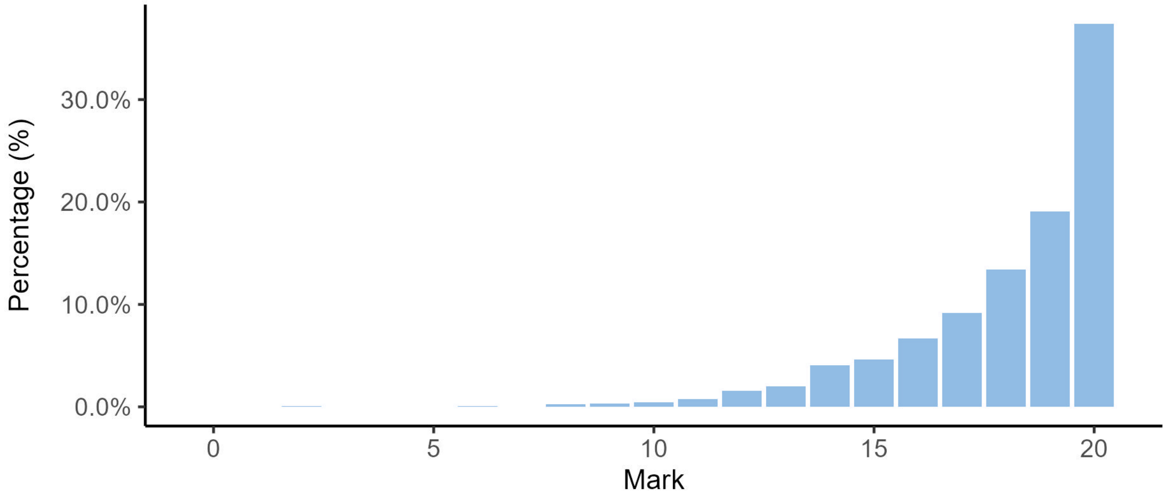
Units 3 and 4 internal assessment (IA) results

Total marks for IA

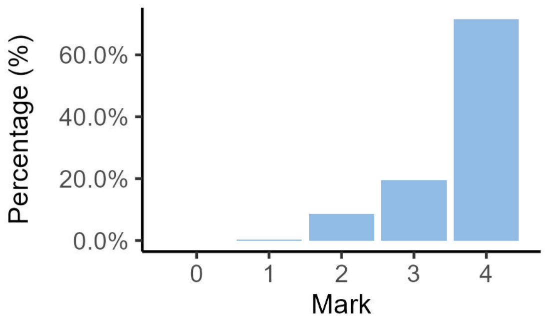


IA1 marks

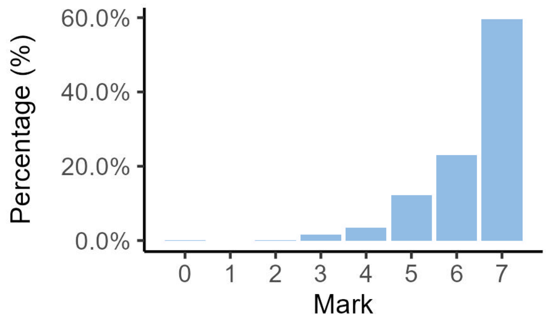
IA1 total



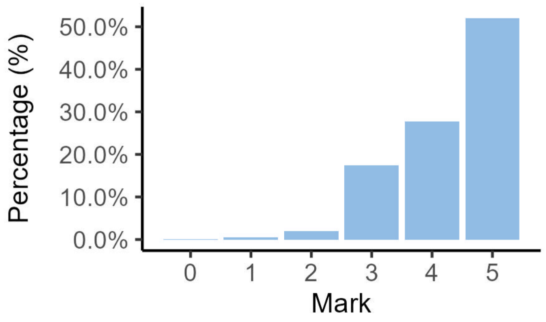
IA1 Criterion: Formulate



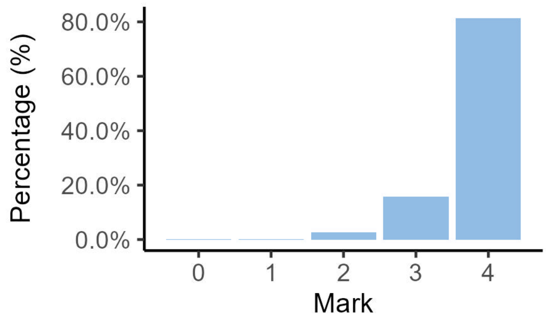
IA1 Criterion: Solve



IA1 Criterion: Evaluate and verify

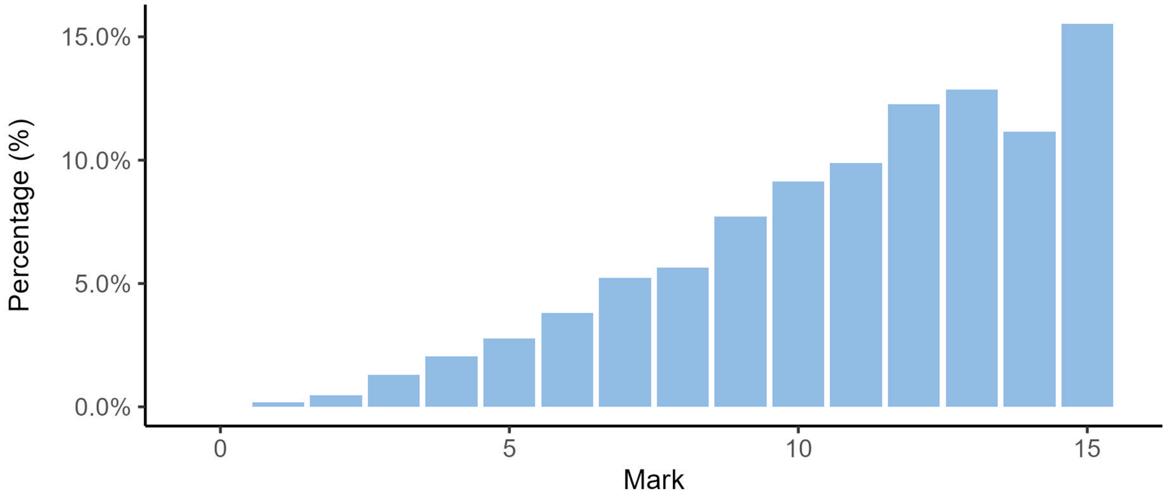


IA1 Criterion: Communicate

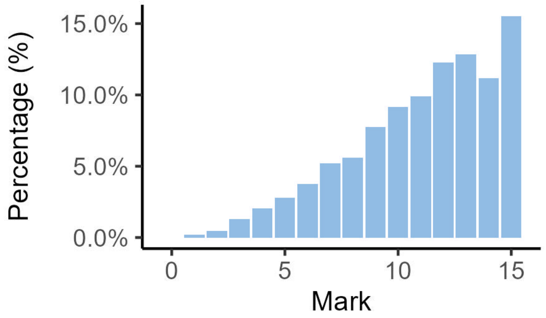


IA2 marks

IA2 total

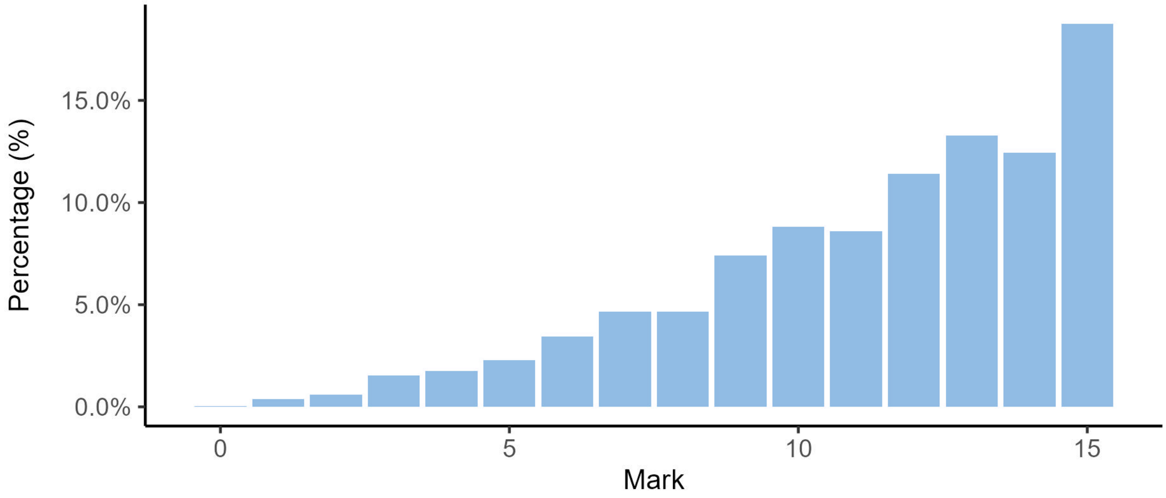


IA2 Criterion: Foundational knowledge and problem-solving

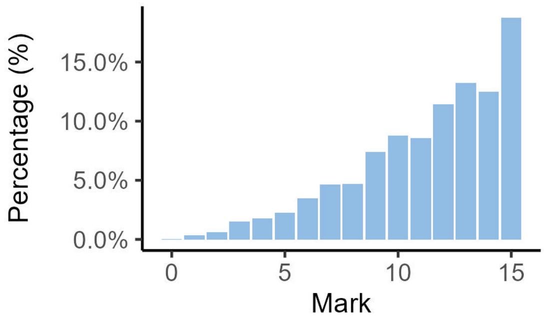


IA3 marks

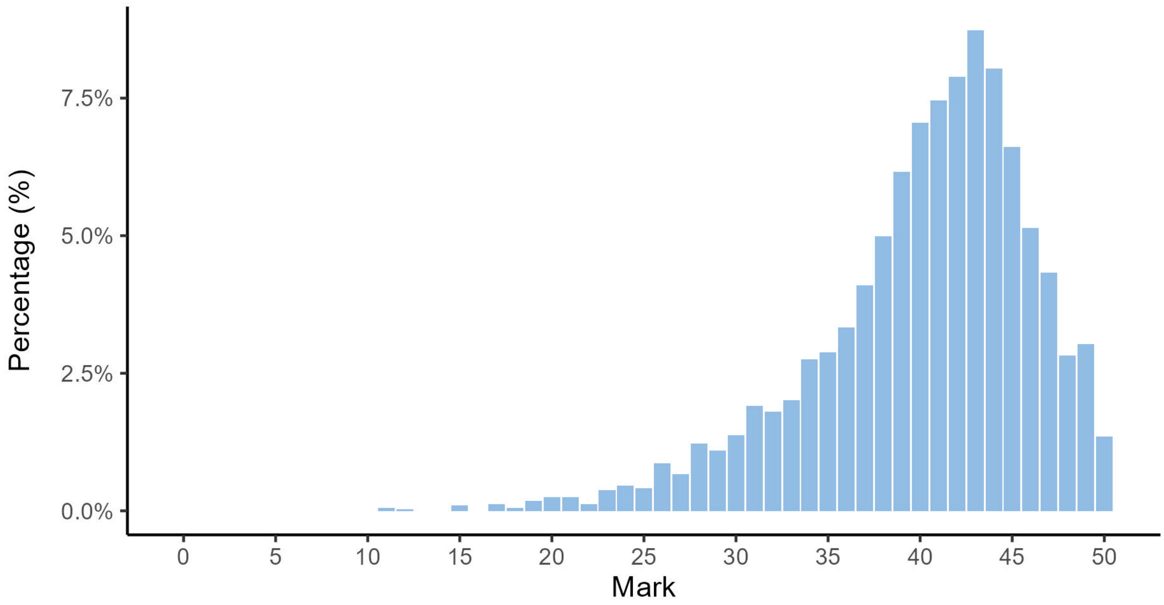
IA3 total



IA3 Criterion: Foundational knowledge and problem-solving

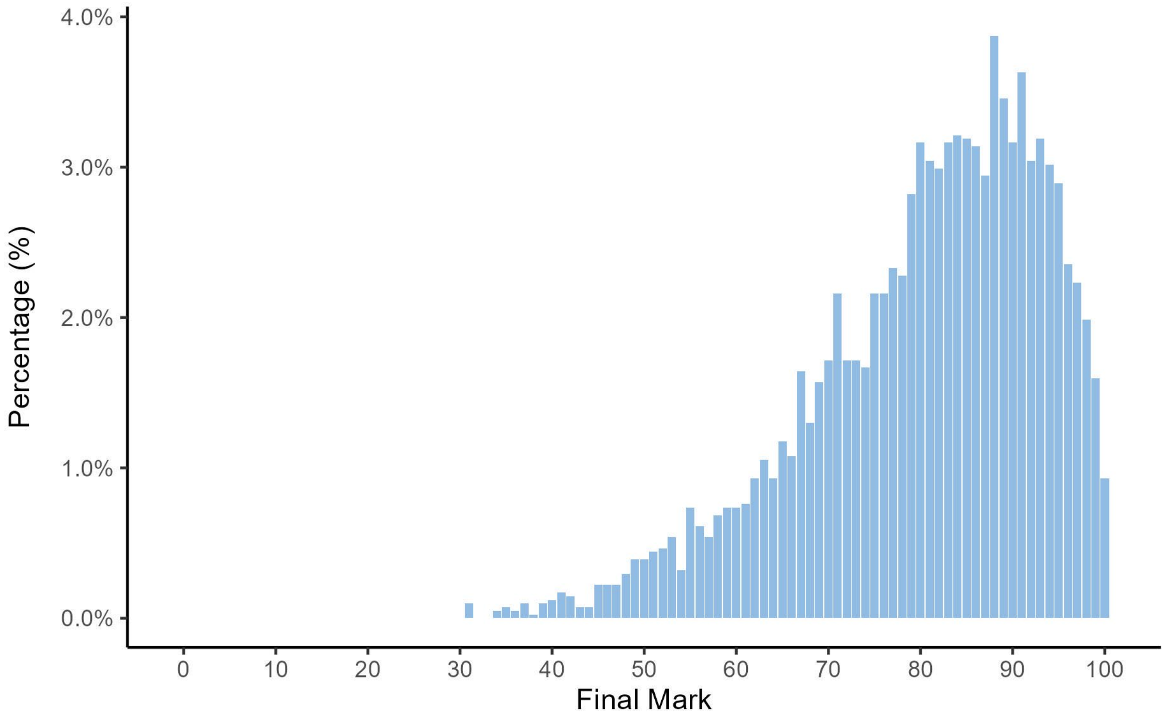


External assessment (EA) marks



Final subject results

Final marks for IA and EA



Grade boundaries

The grade boundaries are determined using a process to compare results on a numeric scale to the reporting standards.

Standard	A	B	C	D	E
Marks achieved	100–86	85–70	69–49	48–22	21–0

Distribution of standards

The number of students who achieved each standard across the state is as follows.

Standard	A	B	C	D	E
Number of students	1,690	1,610	695	83	0

Internal assessment

The following information and advice relate to the assessment design and assessment decisions for each IA in Units 3 and 4. These instruments have undergone quality assurance processes informed by the attributes of quality assessment (validity, accessibility and reliability).

Endorsement

Endorsement is the quality assurance process based on the attributes of validity and accessibility. These attributes are categorised further as priorities for assessment, and each priority can be further broken down into assessment practices.

Data presented in the Assessment design section identifies the reasons why IA instruments were not endorsed at Application 1, by the priority for assessment. An IA may have been identified more than once for a priority for assessment, e.g. it may have demonstrated a misalignment to both the subject matter and the assessment objective/s.

Refer to *QCE and QCIA policy and procedures handbook v6.0*, Section 9.5.

Percentage of instruments endorsed in Application 1

Instruments submitted	IA1	IA2	IA3
Total number of instruments	307	307	302
Percentage endorsed in Application 1	74	50	44

Confirmation

Confirmation is the quality assurance process based on the attribute of reliability. The QCAA uses provisional criterion marks determined by teachers to identify the samples of student responses that schools are required to submit for confirmation.

Confirmation samples are representative of the school's decisions about the quality of student work in relation to the instrument-specific marking guide (ISMG), and are used to make decisions about the cohort's results.

Refer to *QCE and QCIA policy and procedures handbook v6.0*, Section 9.6.

The following table includes the percentage agreement between the provisional marks and confirmed marks by assessment instrument. The Assessment decisions section of this report for each assessment instrument identifies the agreement trends between provisional and confirmed marks by criterion.

Number of samples reviewed and percentage agreement

IA	Number of schools	Number of samples requested	Number of additional samples requested	Percentage agreement with provisional marks
1	301	1,899	4	95.02
2	298	1,891	0	100.00
3	298	1,871	0	100.00

Internal assessment 1 (IA1)

Problem-solving and modelling task — extended response (20%)

This assessment focuses on the interpretation, analysis and evaluation of ideas and information. It is an independent task responding to a particular situation or stimuli. While students may undertake some research in the writing of the problem-solving and modelling task, it is not the focus of this technique. This assessment occurs over an extended and defined period. Students will use class time and their own time to develop a response.

The problem-solving and modelling task must use subject matter from one or both of the following topics in Unit 3:

- Topic 2: Vectors and matrices
- Topic 3: Complex numbers 2.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions
Alignment	44
Authentication	9
Authenticity	13
Item construction	8
Scope and scale	22

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided datasets or used an open-ended task design, which allowed students to develop a unique response
- included relevant stimulus material, where appropriate.

Practices to strengthen

It is recommended that assessment instruments:

- avoid identifying concepts or techniques that lead students to a particular solution, to allow students to independently demonstrate simple through to complex procedures, and to make decisions about the data to be used, what model to develop, and what technology and mathematical techniques are relevant to solve the problem

- avoid scaffolding in the approach to problem-solving and modelling stimulus to allow students to demonstrate their knowledge and understanding of the task criteria.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions
Bias avoidance	1
Language	13
Layout	1
Transparency	3

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- included a reference to the approach to problem-solving and mathematical modelling from Syllabus section 1.2.4 or provided an appropriate task-specific approach.

Practices to strengthen

It is recommended that assessment instruments:

- use clear, concise language and avoid jargon.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Formulate	97.67	2.33	0	0
2	Solve	98.01	1.00	1	0
3	Evaluate and verify	98.67	1.33	0	0
4	Communicate	99.00	0.00	1	0

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- clear distinction was made between observations (data or information required to solve a mathematical problem and/or develop a mathematical model) and assumptions (conditions that are stated to be true) used in the mathematisation process
- teachers clearly identified where students have provided clear documentation of both strengths and limitations relevant to the final model or solution.
- clear documentation was presented of both strengths and limitations relevant to the final model or solution by providing evidence about why decisions and refinements contributed to its success or otherwise. This was different from considering the reasonableness of the result.

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- teachers clearly identify where student evaluation of the reasonableness of solutions consider observations and assumptions specifically targeting those that are considered during formulation.

Samples

The following excerpt has been included as an example of a response where assumptions and observations are clearly identified.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

Considerations:

Documentation of observations

Observations

- The kangaroos species sampled to obtain data was unknown (Species-X), so external data about specific species interacting with this data was minimised.
- A danger threshold of 1.2 kangaroos per square kilometre exists. Below this number, the population could be at risk of short or long term harm (Brooks, 2022).
- A recommended minimum of 100 individuals exists in a given population to combat inbreeding and a minimum of 1000 individuals to prevent genetic drift (Bradshaw, 2014).
- All adults of Species-X die after living for 15 years, as the survival rate for this age group was 0. Therefore, a 15x15 matrix was used and this was pertinent as it impacted the scope of the mathematical model.

Assumptions

- The ratio between males and females of Species-X was assumed to be the same as the Western Grey species - with a female to male ratio of 100:46. This allowed total populations to be determined (Mjadwesch, 2011).
- The population of kangaroos was assumed to exclude migration, enabling the Leslie Matrix to remain valid.
- It was assumed that survival rates and fecundity rates remained constant, enabling the Leslie Matrix to be valid.
- Environmental and ecological factors were assumed to be static, otherwise the survival and fecundity rates could vary, rendering the model unreliable.
- Disease and unpredictable health factors were assumed to not affect survival and fecundity rates, ensuring consistency to maintain the validity of the model.
- Species-X was assumed as a unique species. Therefore, incomplete data was calculated using averages from adjacent regions, keeping data closed to external sources to maintain reliability.
- The survival and fecundity rates were assumed to have incorporated the culling factor used by the Kangaroo Industry and this rate remained fixed, allowing for simpler modelling as an externally sourced culling factor was not required.

Documentation of assumptions

The following excerpt has been included as an example of clear documentation of both strengths and limitations.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

Strengths

- Technology was used for calculations and graphing purposes. This minimises the possibility of human error, allowing for high degrees of accuracy, particularly as decimal place rounding is standardised..
- An adequate solution was identified, which fulfilled task requirements.
- The method and models can be manipulated to be viable for other population growth investigations of different species.
- While growth rate was the focus of this task, the Leslie's matrices can be used to determine future population numbers. Additionally, the population and growth rate can be estimated for further periods by applying the same calculations.

Commented [CY8]: Documented strengths

Limitations

- There are two camel species, Dromedary and Bactrian, which have varying birth and survival rates, due to factors such as gestation period (Bell, 2023). Additionally, Bactrian camels are critically endangered, whereas dromedary camels are largely abundant (Camel Fact Sheet, 2020). Therefore, the model cannot be generalised for all Australian camels, and the control strategy should only target Dromedary camels.
- While the initial model was based off of birth and survival rate guidelines of age groups relative to each other, there were no actual figures identified. The use of arbitrary numbers may be an inaccurate representation of current rates, meaning the control strategy is unusable and ineffective.
- Despite the control strategy, the population still grows exponentially. This leads to overpopulation concerns, as carrying capacity, or the maximum population that can be supported by environmental resources available, is not considered (Sheldon, 2022). Therefore, the population growth identified may be impossible due to environmental limitations, reducing the validity.

Commented [CY9]: Documented limitations

The following excerpt has been included as an example of evaluating the reasonableness of the solution by considering the results, observations and assumptions.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

Evaluation

Reasonableness of solution

Considering results

The solution identified was somewhat reasonable.

Reasonable:

- A population-time graph was created which verified the rate of change identified (Figure 13). Although it was only accurate to 3 decimal places, this is insignificant due to rounding errors. Additionally, the R^2 value was 1, indicating high accuracy.
- Although there were rounding issues, the population growth was verified by determining the age-specific rate, which displayed high degrees of similarity with previously calculations (Table 8).
- A refined model was created which satisfied the original uncontrolled population growth (7-8%). An age-specific control strategy was also produced, which controlled growth rate (2-3%)

Unreasonable:

- The age class proportions are heavily skewed towards younger age groups (Figure 12). However, in a real-world context, offspring require high levels of parental care, potentially resulting in a lower survival rate and population due to inadequate older members (Naumann, 1999).

Considering observations
The observations outlined were somewhat reasonable.
Reasonable:

- Leslie's matrix allowed for population growth calculations, while investigating the effects of altering birth, survival and age-specific populations.

Unreasonable:

- Development significantly varies, particularly during maturation, meaning that the average birth and survival rates may be skewed due to considerable variations in members of the same age class, reducing the accuracy of predicted population growth.
- Survival and birth rates cannot be generalised over a large population, as camels inhabit the entirety of the Australia outback. This suggests that the control strategy may be effective in some regions, while decimating or not affecting the population in others, reducing the overall utility.
- In wild populations, the male to female ratio is skewed at 1:50, favouring females, instead of 1:1 (Abdulmohsen & Al-Ekna, 2018). Therefore, there may be insufficient males to sustain the reproduction levels estimated, suggesting that the population growth calculated is exaggerated.

Considering assumptions
The assumptions were partially reasonable.
Reasonable:

- Population growth was rounded to 5 decimal places. While this introduces error to the calculations, this only affects the stabilisation point, as the rate is viable within a broad range (for example 7-8%). This suggests reasonableness, as this wasn't the focus of the task.

Unreasonable:

- Population growth, birth and survival rates, are impacted by numerous factors, meaning that population growth may not follow predicted exponential relationships. Additionally, the control strategy will likely impact external factors such as predator-prey relationships, particularly as camels are gregarious animals. This reduces the models' accuracy, altering the population growth identified.
- Reproduction only directly involves females, but a sufficient population of mature males is still essential and should be considered in the control strategy. Therefore, the population growth should also consider males, meaning that the rate may be unreasonable as the effect of changing initial population is unknown. Additionally, the lack of male offspring is biologically impossible, and is unreasonable as younger members cannot sustain future population growth, suggesting that the population and estimated growth is invalid.
- Birth and survival rates frequently fluctuate over time, suggesting that population growth is unlikely to follow the exponential trend identified, and is therefore inaccurate.

The following excerpts have been included as excerpt 1 demonstrates the documentation of assumptions and observations and excerpt 2 uses these assumptions and observations when evaluating the strengths and limitations of the solution.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

Excerpt 1

Assumptions + Observations:

Assumptions:

1. It was assumed that migration does not occur within Iraq as the Leslie matrix does not account for it.
2. Resources (males, food, healthcare, money, etc.) were assumed to be unlimited. Apart from war, other existential threats (pandemics, natural disasters, etc.) did not impact Iraq's population as Leslie matrices do not account for such factors (Wikipedia, 2023). Therefore, Iraq's population only increased due to births and decreased due to the impacts of war and survival rates as these were the only factors considered.
3. The birthrate and survival rate were assumed to remain constant as a Leslie matrix cannot adjust values accordingly with the realistic rates (ISSUU, 2017).
4. The entire Iraq population undergo the same conditions as two values (birth and survival rate) describe an entire age-group.
5. It was assumed that the men and women survival rates were equal, given they undergo identical conditions.

Observations:

1. The ratio between males and females is approximately 1:1 (INED, 2020).
2. The Leslie matrix only considers female population as they were the ones who give birth and will have data, whereas the father can be unknown (LibreTexts, 2021).
3. Women aged 50+ and under 15 have extremely minimal fertility rates within Iraq, where it can be considered negligible. (British Fertility Society, 2018 & Schoumaker, B., Sánchez-Páez, D., 2022).

Excerpt 2

Evaluation:

Strengths:

1. Both of the Leslie matrices were quite reasonable and accurate in the population prediction as signified by the accuracy evaluation, where neither matrix exceed the threshold of 10% apart from the low population age-groups.
2. The data used to model the matrices was incredibly reliable as it was from the United Nations, a renowned worldwide organisation. Therefore, data collection did not present any implications.
3. The consideration of the female population only for the Leslie matrix was reasonable due to observation 2.
4. The 5-year age-group interval was deemed slightly reasonable as it was shorter than major life stages and also prevented any significant fluctuations. However, fluctuations do still occur, and to minimise this, single-age-groups could be adapted.

Limitations & Improvements:

1. The pre-war matrix data was from 1975-1979, whereas the war data was from 1980-1984, which introduced error as conditions apart from war were not identical. This can be improved by using data from the same period.
2. Assumption 1 was unreasonable as it was disproved by several sources (*Iraq - Migrants & Refugees Section*, 2020). To improve this, other matrix methods would have to be used as Leslie matrices do not account for migration.
3. The eigenvalue calculations were subject to error, as the determinant was not exactly zero, and therefore the eigenvalue cannot represent the exact growth rate of either matrix.
4. Assumptions 2, 3 and 4 were unreasonable due the constant fluctuation of socio-economic conditions and differing living standards. This would result in an inconsistent birthrate and survival rate and other factors such as money and hunger stunting and affecting the Iraqi population growth differently in different areas (Romaszko, J., et al., 2017). This can be minimised by increasing the sample size, which would reduce the effect of these fluctuations and thus increasing accuracy. The different rates in different demographics can be reduced by analysing a more specific demographic, where majority of the population have near-identical living standards, as Iraq was described to have a 25% poverty rate (themedialine, 2023).

Additional advice

- It is recommended that teachers annotate assessment responses as annotations help identify the evidence they have used to determine where students have met descriptors from the ISMG.
- It is recommended that care is taken with identifying documentation as student-provided headings do not always align with student-provided statements. Documentation of statements of assumptions, observations, strengths and limitations require citations, written references or decisive information to support the statements.

Internal assessment 2 (IA2)

Examination — short response (15%)

The examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 3 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions
Alignment	116
Authentication	0
Authenticity	1
Item construction	11
Scope and scale	25

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- assessed a representative sample of knowledge and skills from all topics in Unit 3 (AS unit 3) in well-constructed questions that reflected the intended learning of the subject matter
- constructed complex familiar questions without scaffolding by avoiding splitting these questions into parts a), b), etc., as splitting reduces the opportunity for the clarification and analysis required to develop responses to items of this degree of difficulty
- included items that assessed Assessment objective 4: evaluate the reasonableness of solutions, and assigned corresponding marks in the marking scheme.

Practices to strengthen

It is recommended that assessment instruments:

- include complex unfamiliar questions that match the syllabus description for both complexity and unfamiliarity (Syllabus section 4.6.2), i.e. relationships and interactions have a number of elements, and all the information to solve the problem is not immediately identifiable
- do not use standalone questions that only assess the review of assumed knowledge of subject matter from Unit 1, Unit 2 (AS unit 1, AS unit 2) or Mathematical Methods
- provide the opportunity for students to complete the examination in the prescribed time.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions
Bias avoidance	1
Language	29
Layout	4

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- clearly stipulated use of technology or an algebraic approach for questions where examinations were not separated into technology-free and technology-active papers.

Practices to strengthen

It is recommended that assessment instruments:

- provide cues such as 'using a vector calculus approach' when the question can be solved using non-Specialist Mathematics techniques.

Additional advice

- Ensure that the marking scheme matching the assessment instrument is checked for accuracy and uploaded for the endorsement review. The marking scheme should demonstrate the match of the instrument to assessment specifications and show the distribution of mark allocations, which helps schools check the scope and scale of the assessment, including such aspects as time management, adequacy of response space, and alignment with the intended degree of difficulty.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	100	0	0	0

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- schools clearly indicated on the student response which evidence was used to award marks and there was alignment between the marks awarded and the marking scheme
- the total mark awarded for each question was indicated on the student response
- the total number of marks awarded, the percentage calculated, and the provisional ISMG mark were clearly annotated on the ISMG
- any errors in student responses that were later used in valid procedures were clearly annotated to indicate how follow-through marks had been awarded.

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- if a response shows an acceptable alternative solution to the one shown in the marking guide, the school clearly annotates on the student response how or why the decision to award the mark has been made.

Samples

The following excerpts demonstrate clear indication on the student response where the marks were awarded, alignment with the marking scheme and each question's total awarded marks.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

Excerpt 1

Question 5 (6.5 marks) Mark/s: $6\frac{1}{2}$

When $n=1$, LHS: $(\cos\theta + i\sin\theta)^1$ RHS: $\cos(1\times\theta) + i\sin(1\times\theta)$
$= \cos\theta + i\sin\theta$ $= \cos\theta + i\sin\theta$ $\therefore$ true for first term ✓

Assume true for $n=k$,
$(\cos\theta + i\sin\theta)^k = \cos(k\theta) + i\sin(k\theta)$ ✓

When $n=k+1$,
$(\cos\theta + i\sin\theta)^{k+1} = \cos((k+1)\theta) + i\sin((k+1)\theta)$ ✓

LHS:
$(\cos\theta + i\sin\theta)^{k+1}$
$= (\cos\theta + i\sin\theta)^k \times (\cos\theta + i\sin\theta)^1$ ✓
$= (\cos(k\theta) + i\sin(k\theta)) \times (\cos\theta + i\sin\theta)$ ✓
$= \cos(k\theta)\cos\theta + \cos(k\theta)i\sin\theta + i\sin(k\theta)\cos\theta + i^2\sin(k\theta)\sin\theta)$
$= \cos(k\theta)\cos\theta - \sin(k\theta)\sin\theta + i(\sin(k\theta)\cos\theta + \cos(k\theta)\sin\theta)$
$= \cos(k\theta+\theta) + i\sin(k\theta+\theta)$
$= \cos((k+1)\theta) + i\sin((k+1)\theta)$ ✓
$=$ RHS
QED

As the statement is true for $n=1$, and is true for $n=k+1$ when $n=k$ is true, the statement is proven true for $n \in \mathbb{Z}^+$ by mathematical induction. $\frac{1}{2}$

Excerpt 2

Question 15 (5 marks) Mark/s: 5

Use knowledge of complex numbers demonstrate that

$$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

for all values of A and B.

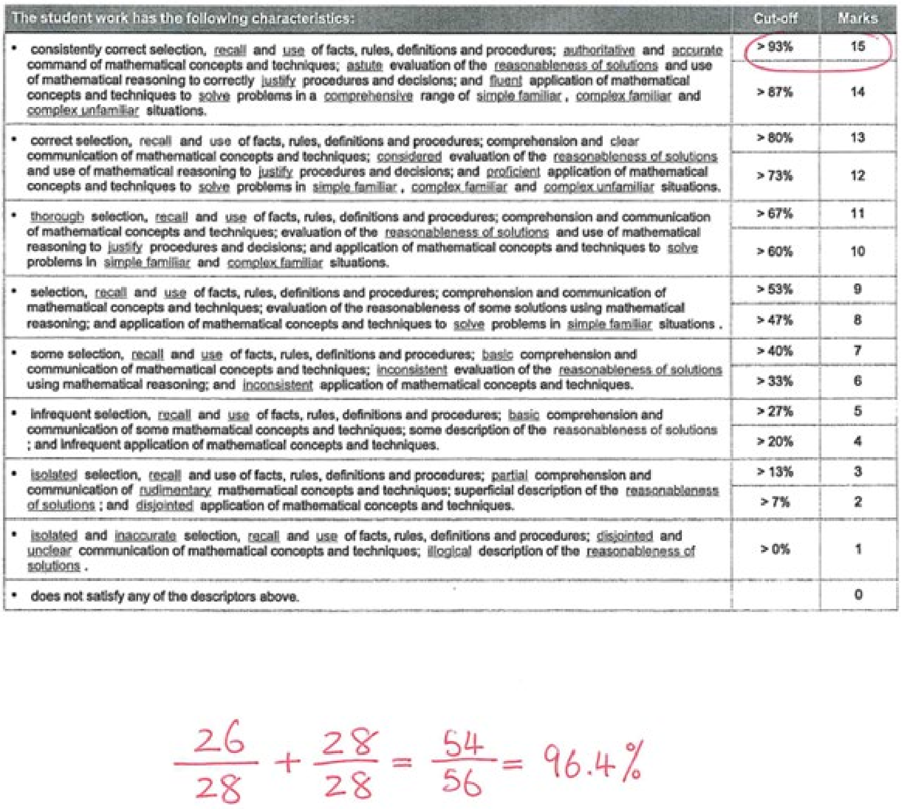
let $z = \text{cis}(A)$, $w = \text{cis}(B)$ ✓✓
$z \times w = (\text{cis}(A))(\text{cis}(B))$
$= (\cos(A) + i\sin(A))(\cos(B) + i\sin(B))$
$= \cos(A)\cos(B) + i\cos(A)\sin(B) + i\sin(A)\cos(B) + i^2\sin(A)\sin(B)$
$= \cos(A)\cos(B) + i\cos(A)\sin(B) + i\sin(A)\cos(B) - \sin(A)\sin(B)$ ✓✓
$\text{Im}(z\times w) = \sin(A)\cos(B) + \cos(A)\sin(B)$
via de moivre theorem $z \times w = \text{cis}(A+B)$
$\text{cis}(A+B) = \cos(A+B) + i\sin(A+B)$ ✓✓
$\text{Im}(z\times w) = \sin(A+B)$

$\text{Im}(z\times w) = \text{Im}(z\times w)$
$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
This proves that $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
for all values of A and B. ✓✓

✓✓ logic.

The following excerpt has been included as an example of an annotated ISMG with the total number of marks awarded, the percentage calculated and the provisional ISMG mark clearly identified.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

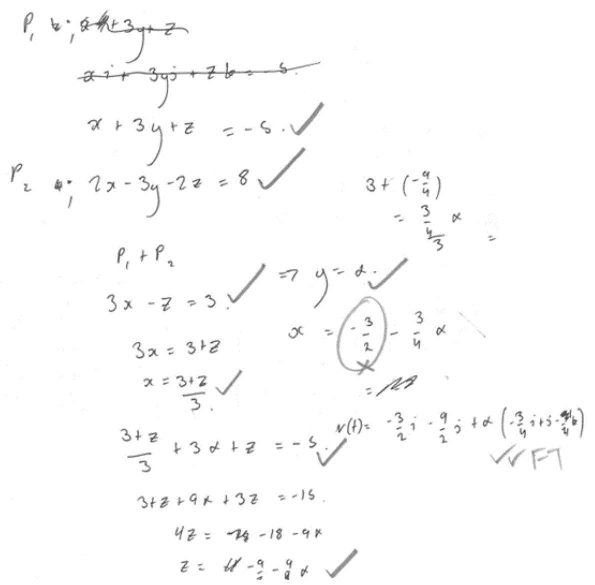


The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations .	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions ; and infrequent application of mathematical concepts and techniques.	> 27%	5
	> 20%	4
• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions ; and disjointed application of mathematical concepts and techniques.	> 13%	3
	> 7%	2
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; illogical description of the reasonableness of solutions .	> 0%	1
• does not satisfy any of the descriptors above.		0

$\frac{26}{28} + \frac{28}{28} = \frac{54}{56} = 96.4\%$

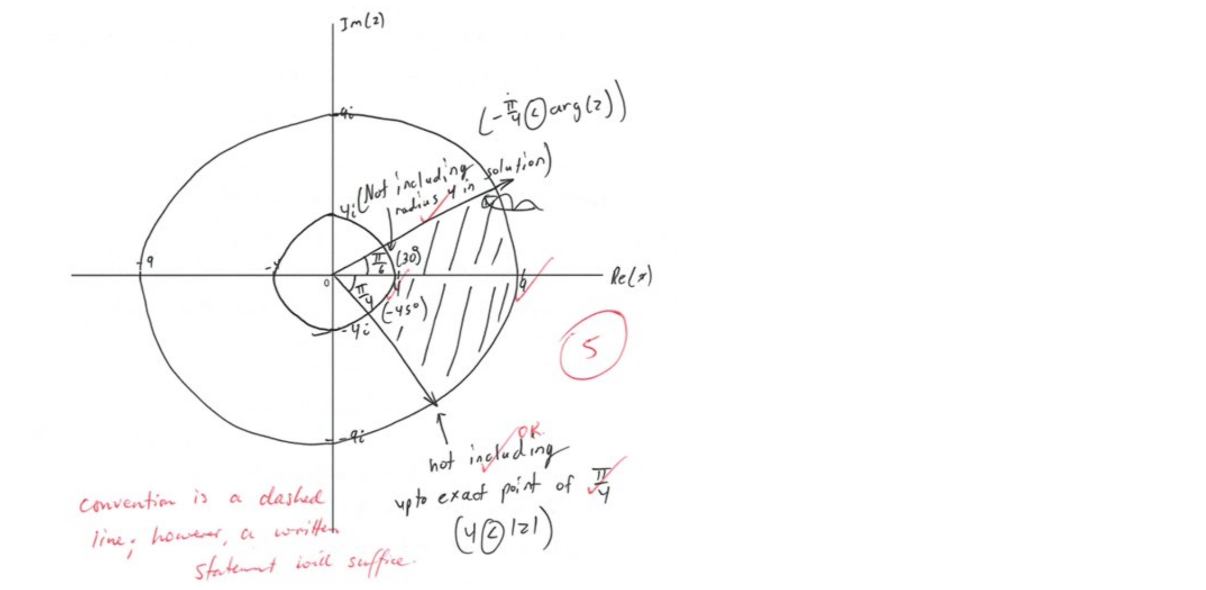
The following excerpt has been included as an example of an error in a student response that was clearly identified and where the use of follow-through marks was clear.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.



The following excerpt has been included as an example of an acceptable alternative solution annotated to indicate the reason marks were awarded.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.



Additional advice

- To ensure the accuracy of judgments, schools are advised to apply internal quality assurance processes to check marking decisions, raw mark totals and the application of percentage cut-offs to award the ISMG mark out of 15.
- Schools must use endorsed assessment. If changes need to be made after endorsement, the QCAA amendment process must be followed. (See *QCE and QCIA policy and procedures handbook v6.0*, Section 9.5.6 Amending an endorsed assessment instrument (whole cohort).) If changes need to be made during assessment, it will be necessary to contact the subject PEO. The marking scheme can be updated after students have completed the assessment. However, there must not be changes to the number of marks allocated to questions, or to the classification of questions.
- In the *Developing summative internal assessment instruments: Endorsement user guide* available in the help section for the Endorsement application (app), one of the examples for the instructions states students should write their answers in the response booklet using black or blue pen. When students use pencil, care must be taken to avoid poor file quality because the response may be faint or unreadable. When students write responses on separate lined or blank paper, care must be taken to avoid file errors due to work being out of order, pages missed, or work lost that was done on the back of the page or on the question paper.
- In order for partially correct responses to be awarded marks consistently across the cohort, the marking scheme should indicate the specific skills, steps or expectations required to be awarded the marks.
- If students have been awarded marks for implied working, i.e. working that was not explicitly stated but can be evidenced by subsequent work, this should be annotated on the student script.

Internal assessment 3 (IA3)

Examination — short response (15%)

This examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 4 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions
Alignment	145
Authentication	0
Authenticity	1
Item construction	19
Scope and scale	15

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- assessed a representative sample of knowledge and skills from all topics in Unit 3 (AS unit 3) in well-constructed questions that reflected the intended learning of the subject matter
- constructed complex familiar questions without scaffolding by avoiding splitting these questions into parts a), b), etc., as splitting reduces the opportunity for the clarification and analysis required to develop responses to items of this degree of difficulty
- included items that assessed Assessment objective 4: evaluate the reasonableness of solutions and assigned corresponding marks in the marking scheme
- assessed subject matter from Unit 4 Specialist Mathematics (AS unit 4) only, especially in the topic of Statistical inference.

Practices to strengthen

It is recommended that assessment instruments:

- include complex unfamiliar questions that match the syllabus description for both complexity and unfamiliarity (Syllabus section 5.6.1), i.e. relationships and interactions have a number of elements, and all the information to solve the problem is not immediately identifiable
- do not use standalone questions that only assess the review of assumed knowledge of subject matter from Unit 1, Unit 2 or Unit 3 (AS unit 1, AS unit 2 or AS unit 3) or Mathematical Methods

- provide the opportunity for students to complete the examination in the prescribed time.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions
Bias avoidance	2
Language	28
Layout	9
Transparency	20

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- clearly stipulated use of technology or an algebraic approach for questions where examinations were not separated into technology-free and technology-active papers.

Practices to strengthen

It is recommended that assessment instruments:

- provide cues such as ‘using a calculus approach’ when the question can be solved using non-Specialist Mathematics techniques.

Additional advice

- Ensure that the marking scheme matching the assessment instrument is checked for accuracy and uploaded for the endorsement review. The marking scheme should demonstrate the match of the instrument to assessment specifications and show the distribution of mark allocations, which helps schools check the scope and scale of the assessment, including such aspects as time management, adequacy of response space, and alignment with the intended degree of difficulty.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	100	0	0	0

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- schools clearly indicated on the student response which evidence was used to award marks and there was alignment between the marks awarded and the marking scheme
- the total mark awarded for each question was indicated on the student response
- the total number of marks awarded, the percentage calculated, and the provisional mark were clearly annotated on the ISMG
- any errors in student responses that were later used in valid procedures were clearly annotated to indicate how follow-through marks had been awarded.

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- if a response shows an acceptable alternative solution to the one shown in the marking guide, the school clearly annotates on the student response how or why the decision to award the mark has been made.

Samples

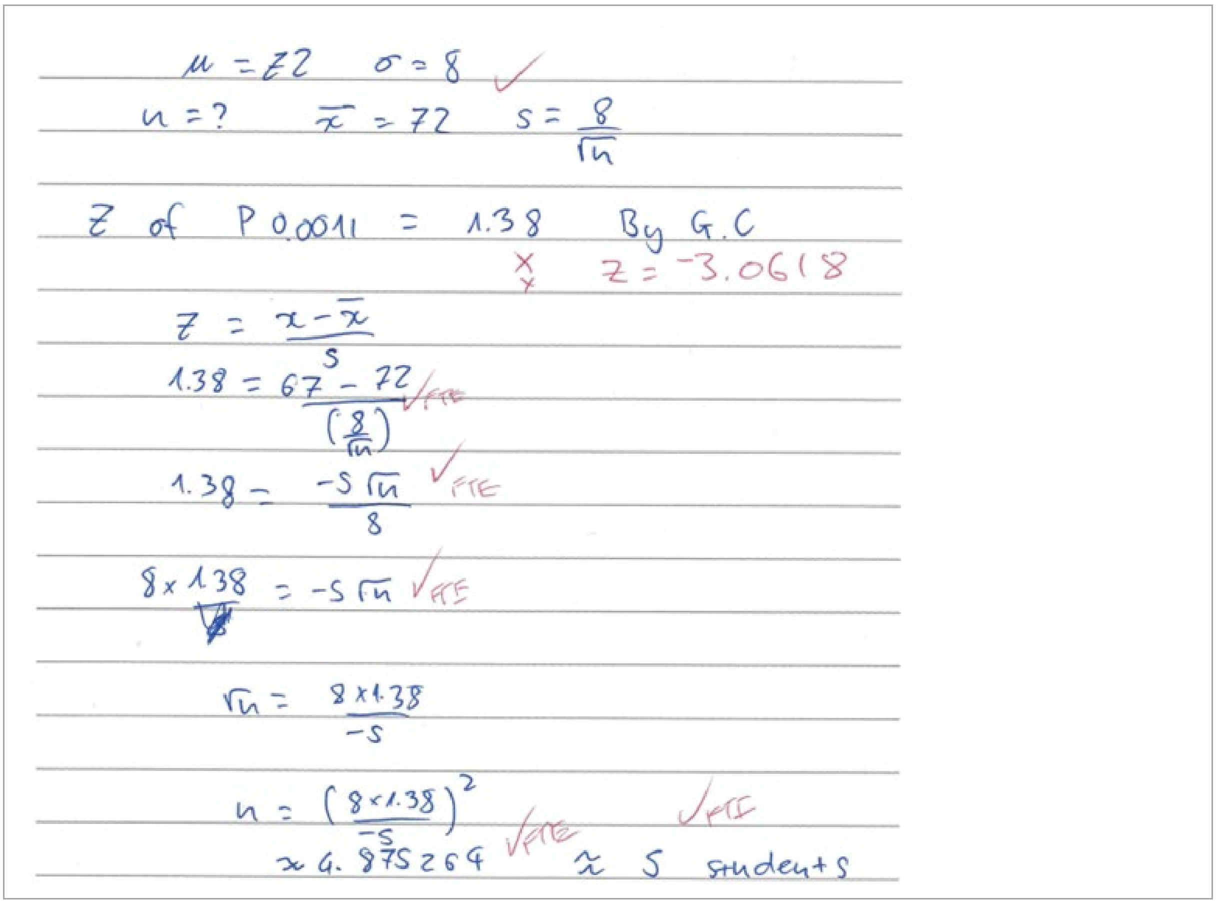
The following excerpt illustrates recording the total number of marks awarded and the corresponding percentage calculation.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

Totals	39 39	13 11.5	13 12	65 62.5
Percentage	60%	20%	20%	100% 96.1 %

The following excerpt has been included as an example of a response where errors in the student response were later used in valid procedures and it is clear how follow-through marks were awarded.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.



$\mu = 72 \quad \sigma = 8$ ✓

$n = ? \quad \bar{x} = 72 \quad s = \frac{8}{\sqrt{n}}$

Z of P 0.0011 = 1.38 By G.C

✗ $z = -3.0618$

$z = \frac{x - \bar{x}}{s}$

$1.38 = \frac{67 - 72}{\left(\frac{8}{\sqrt{n}}\right)}$ ✓FTE

$1.38 = \frac{-5\sqrt{n}}{8}$ ✓FTE

$8 \times 1.38 = -5\sqrt{n}$ ✓FTE

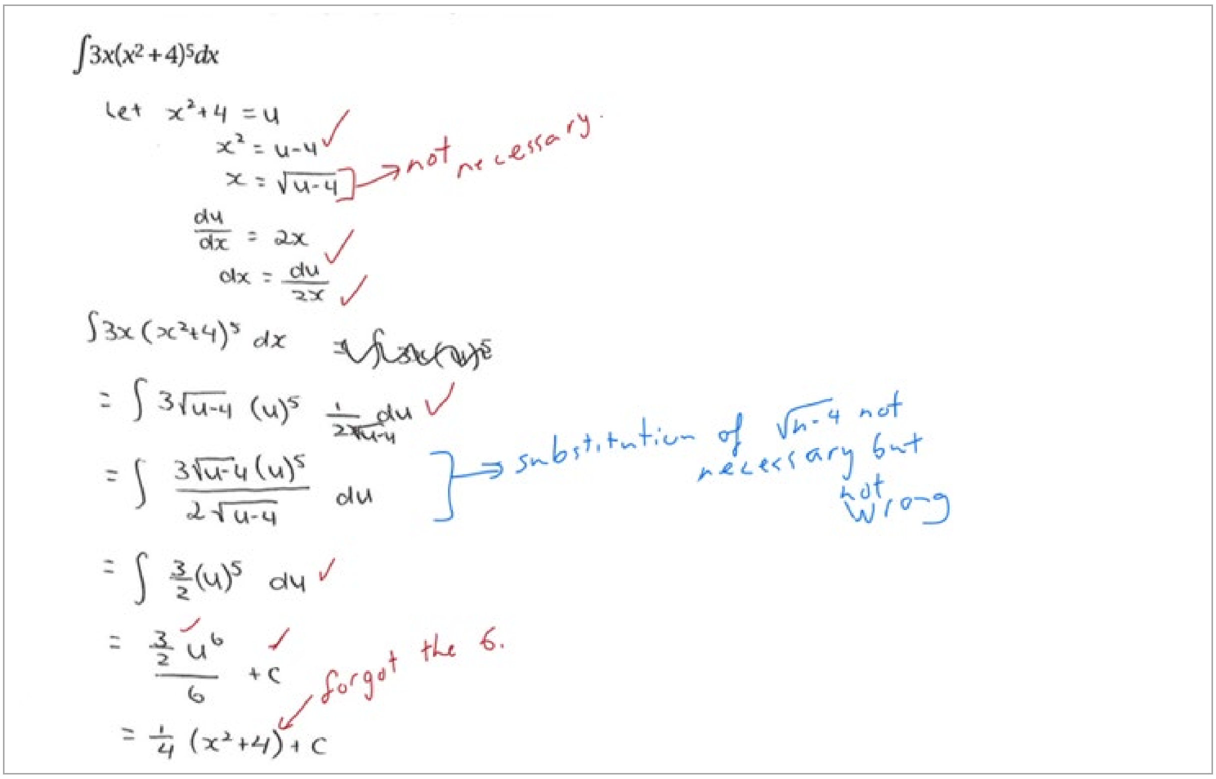
$\sqrt{n} = \frac{8 \times 1.38}{-5}$

$n = \left(\frac{8 \times 1.38}{-5}\right)^2$ ✓FTE

≈ 4.875264 ✓FTE ≈ 5 students ✓FTE

The following excerpt has been included as an example of an acceptable alternative solution annotated to indicate the reason marks were awarded.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.



$\int 3x(x^2+4)^5 dx$

Let $x^2 + 4 = u$ ✓

$x^2 = u - 4$ ✓

$x = \sqrt{u-4}$ → not necessary.

$\frac{du}{dx} = 2x$ ✓

$dx = \frac{du}{2x}$ ✓

$\int 3x(x^2+4)^5\, dx$

$= \int 3\sqrt{u-4}\,(u)^5 \frac{1}{2\sqrt{u-4}} du$ ✓

$= \int \frac{3\sqrt{u-4}\,(u)^5}{2\sqrt{u-4}} du$ → substitution of $\sqrt{u-4}$ not necessary but not wrong

$= \int \frac{3}{2}(u)^5\, du$ ✓

$= \frac{\frac{3}{2}u^6}{6} + c$ ✓ forgot the 6.

$= \frac{1}{4}(x^2+4) + c$

Additional advice

- To ensure the accuracy of judgments, schools are advised to apply internal quality assurance processes to check marking decisions, raw mark totals and the application of percentage cut-offs to award the ISMG mark out of 15.
- Schools must use endorsed assessment. If changes need to be made after endorsement, the QCAA amendment process must be followed. (See *QCE and QCIA policy and procedures handbook v6.0*, Section 9.5.6. Amending an endorsed assessment instrument (whole cohort).) If changes need to be made during assessment, it will be necessary to contact the subject PEO. The marking scheme can be updated after students have completed the assessment. However, there must not be changes to the number of marks allocated to questions, or to the classification of questions.
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- In order for partially correct responses to be awarded marks consistently across the cohort, the marking scheme should indicate the specific skills, steps or expectations required to be awarded the marks.
- If students have been awarded marks for implied working, i.e. working that was not explicitly stated but can be evidenced by subsequent work, this should be annotated on the student script.

External assessment

External assessment (EA) is developed and marked by the QCAA. The external assessment for a subject is common to all schools and administered under the same conditions, at the same time, on the same day.

Examination — short response (50%)

Assessment design

The assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the syllabus.
The examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (50 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (50 marks).

The alternative sequence (AS) assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the AS. The AS examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (50 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (50 marks).

Assessment decisions

Assessment decisions are made by markers by matching student responses to the external assessment marking guide (EAMG). The external assessment papers and the EAMG are published in the year after they are administered.

Multiple choice question responses

There were 10 multiple choice questions in Paper 1.

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Question	A	B	C	D
1	**72.72**	4.32	2.83	19.72
2	2.65	**89.23**	5.14	2.60
3	**88.41**	8.33	2.60	0.49
4	12.70	**76.40**	5.42	5.19
5	3.08	1.75	2.34	**92.65**
6	3.73	**93.11**	1.44	1.54
7	25.78	6.20	18.71	**48.74**
8	**72.90**	2.57	4.52	19.72
9	5.55	3.16	**79.43**	11.54
10	7.28	11.59	**68.35**	12.34

There were 10 multiple choice questions in Paper 2.

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Question	A	B	C	D
1	**84.90**	3.72	9.12	1.93
2	0.67	4.57	**94.37**	0.28
3	4.93	4.73	**82.94**	7.14
4	2.77	4.65	3.16	**89.11**
5	10.45	**75.42**	5.99	7.86
6	7.99	**69.07**	19.09	2.77
7	10.51	**30.31**	34.34	23.81
8	4.08	17.06	8.89	**69.30**
9	6.94	7.63	8.09	**76.65**
10	6.34	3.85	**72.13**	17.13

There were 10 multiple choice questions in Paper 1 (AS).

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Question	A	B	C	D
1	**84.14**	4.14	2.76	8.97
2	9.66	**82.07**	0.69	7.59
3	20.00	1.38	**75.17**	3.45
4	10.34	4.83	43.45	**41.38**
5	38.62	**47.59**	3.45	9.66
6	4.83	**90.34**	2.07	2.76
7	8.97	9.66	**59.31**	21.38
8	**62.07**	4.14	6.21	27.59
9	12.41	20.69	15.86	**50.34**
10	10.34	8.28	**61.38**	20.00

There were 10 multiple choice questions in Paper 2 (AS).

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Question	A	B	C	D
1	**82.07**	4.14	10.34	3.45
2	1.38	9.66	**88.97**	0.00
3	2.07	**69.66**	8.97	18.62
4	4.83	3.45	6.90	**84.83**
5	**61.38**	13.79	20.69	3.45
6	9.66	**63.45**	20.69	4.83
7	8.97	**30.34**	42.76	17.93
8	4.14	23.45	14.48	**57.24**
9	9.66	13.10	**66.90**	9.66
10	14.48	6.90	**60.00**	17.93

Effective practices

Overall, students responded well to:

- the opportunity to demonstrate knowledge and understanding of matrix algebra within the solution of a matrix equation, including the assumed knowledge of calculations involving the inverse and matrix multiplication of matrices of dimension 3, using technology
- the opportunity to demonstrate knowledge and understanding of the development of a slope field and algebraic solution from a given differential equation
- the opportunity to demonstrate the use of technology in calculating approximate confidence intervals for the population mean, μ, for varying levels of confidence
- multiple choice questions on both papers.

Practices to strengthen

When preparing students for external assessment, it is recommended that teachers consider:

- providing opportunities for students to practise Objective 4: evaluate reasonableness of solutions
- providing opportunities for students to practise converting the equation of a line between parametric, vector and Cartesian forms
- providing opportunities for students to extend the assumed knowledge of concepts of vectors from two-dimensional space to three-dimensional space, e.g. determining a vector in Cartesian form that represents the relative position/velocity of object B from object A (and vice versa) for two objects moving in three dimensions
- providing opportunities for students to compare and contrast the distributions of X and $\bar{X}$, including calculations and notations related to their respective means and standard deviations. Students should also be encouraged to examine the approximate normality of sample mean distributions based on either normally distributed populations or in relation to the application of the central limit theorem where the sample size is large ($n \geq 30$).

Samples

Short response

Paper 1, Question 13

The following excerpt is from Question 13 of Paper 1. This simple familiar question required students to demonstrate an understanding of the remainder theorem in regard to two polynomials $P(z)$ and $Q(z)$ that contain real and imaginary coefficients.

Effective student responses:

- used the remainder theorem to determine a relationship between the two polynomials
- determined simplified expressions for $P(i)$ and $Q(i)$
- determined the required unknown value.

This excerpt has been included:

- to demonstrate suitable use of the remainder theorem
- to show correct calculations involving the complex number i
- to demonstrate the appropriate use of mathematical reasoning to solve the problem.

Remainder theorm $P(a)$ = remainder.

$P(i) = Q(i)$.

$$a(i)^2 - i(i) + 1 - 3i = (i)^2 + 3i(i) + 2a$$

$$-a + 1 + 1 - 3i = -1 - 3 + 2a$$

$$-a + 2 - 3i = -4 + 2a$$

$$-a + 2 - 3i + 4 - 2a = 0$$

$$-3a + 6 - 3i = 0$$

$$3a = 6 - 3i$$

$$a = 2 - i.$$

$\therefore a = 2 - i$ for $P(i)$ and $Q(i)$ to have the same remainder.

Paper 1, Question 16

The following excerpt is from Question 16 of Paper 1. This complex familiar question required students to use mathematical induction to prove a divisibility result.

Effective student responses:

- understood the nature of inductive proof, including the use of an initial statement, assumption statement and inductive step
- used appropriate algebraic procedures to complete the inductive step
- communicated a suitable conclusion to the proof by mathematical induction
- used logical organisation to communicate the proof.

This excerpt has been included:

- to illustrate an example of the establishment of a suitable divisibility proposition based on a multiple of 7
- to demonstrate a proof of the initial statement and inclusion of an appropriate assumption statement
- to demonstrate a proof of the inductive step using knowledge of suitable algebraic procedures
- to demonstrate an appropriate conclusion statement to the proof
- to demonstrate logical organisation by communicating key steps, including clear flow of the required proof and appropriate use of LHS = RHS conventions.

Let $P(n)$ be the proposition that $12^n + 2(5^{n-1}) = 7a$, $a \in \mathbb{Z}$
$\forall n \in \mathbb{Z}^+$

Prove true for $n=1$: i.e. $P(1)$

LHS $= 12^1 + 2(5^{1-1})$

$= 12 + 2\times 5^0 = 12 + 2$

$= 14 = 2(7) =$ RHS $\quad 7 \in \mathbb{Z}$

$\therefore P(1)$ is true.

Assum $P(k)$ is true.

i.e. Assume $12^k + 2(5^{k-1}) = 7b$, $b \in \mathbb{Z}$

RTP: $P(k+1)$ is true. i.e. RTP: $12^{k+1} + 2(5^{k+1-1}) = 7c$, $c \in \mathbb{Z}$

LHS $= 12^{k+1} + 2(5^k)$

$= 12 \times 12^k + 2(5^{k-1} \times 5)$

$= 7\times 12^k + 5\times 12^k + 5\times 2(5^{k-1})$

$= 7\times 12^k + 5(12^k + 2(5^{k-1}))$

$= 7\times 12^k + 5(7b)$ via assumption

$= 7(12^k + 5b)$

$= 7c =$ RHS where $c \in \mathbb{Z}$ since $c = 12^k + 5b$.

$\therefore P(k) \Rightarrow P(k+1)$ is true, and since $P(1)$ is true, by principle of mathematical induction $P(n)$ is true $\forall n \in \mathbb{Z}^+$

that is $12^n + 2(5^{n-1}) = 7a$, $a \in \mathbb{Z}$ $\forall n \in \mathbb{Z}^+$.

Paper 1, Question 19

The following excerpt is from Question 19 of Paper 1. This complex unfamiliar question required students to develop an equation involving complex numbers and evaluate the reasonableness of possible solutions.

Effective student responses:

- established a suitable complex equation based on the given information
- used understanding associated with the argument of a complex number that had been raised to a power
- considered the general nature of solutions to a trigonometric equation
- used the given conditions to evaluate the reasonableness of the solution.

This excerpt has been included:

- to demonstrate the development of an appropriate complex equation based on the given information

- to demonstrate the use of the argument of a complex number in the development of the response
- to demonstrate the general solution of a trigonometric equation
- to demonstrate mathematical reasoning in the sequential development of the response
- to provide evidence of the consideration of the given conditions to evaluate the reasonableness of the solution.

$|w| = \sqrt{x^2+1^2} = \sqrt{x^2+1} = r$

$\arg(w) = \tan^{-1}\left(\frac{1}{x}\right) = \theta$

$\therefore w = \sqrt{x^2+1^2}\ \text{cis}\left(\tan^{-1}\left(\frac{1}{x}\right)\right)$

let $w = r\,\text{cis}(\theta)$

$\therefore w^7 = r^7\,\text{cis}(7\theta)$ by demoivre's theorem

$Re(w^7) = r^7\cos(7\theta)$

for $Re(w^7) = 0$ then ~~$7\theta = k\pi$ where $k \in \mathbb{Z}$~~

$7\theta = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

$\therefore \theta = \frac{\pi}{14} + \frac{k\pi}{7}$

$\theta \in \left(0, \frac{\pi}{2}\right)$

~~$\therefore 7\theta \in \left(0, \frac{7\pi}{2}\right)$~~

$\therefore k = 0, 1, 2$ $k \neq 3 \therefore \theta = \frac{\pi}{2}$ which is not in domain

$\therefore \theta = \frac{\pi}{14}, \frac{3\pi}{14}, \frac{5\pi}{14}.$

$\theta = \tan^{-1}\left(\frac{1}{x}\right)$

$\therefore \frac{1}{x} = \tan(\theta)$

$\therefore x = \frac{1}{\tan(\theta)}$

$\therefore x = \frac{1}{\tan\left(\frac{\pi}{14}\right)}, \frac{1}{\tan\left(\frac{3\pi}{14}\right)}$ & $\frac{1}{\tan\left(\frac{5\pi}{14}\right)}$

Paper 2, Question 17

The following excerpt is from Question 17 of Paper 2. It required students to use a vector calculus approach to prove a result involving the magnitude of acceleration of an object moving with circular motion.

Effective student responses:

- used a vector calculus approach as specified in the problem
- developed the relationship between the speed of the object and its angular velocity, rather than simply stating a formula not specified in the Specialist Mathematics syllabus
- calculated the magnitude of a vector in Cartesian form
- used logical organisation within the communication of the proof.

This excerpt has been included:

- to demonstrate the use of a suitable vector calculus approach to model the velocity and acceleration vectors of the object
- to demonstrate the use of the velocity and acceleration vectors of the object in Cartesian form to determine the corresponding magnitudes of velocity and acceleration
- to model well-constructed reasoning to develop relationships for both the magnitudes of the velocity and acceleration of the object in terms of the angular velocity and radius of the circular motion of the object
- to demonstrate the logical organisation and the communication of key steps, including the appropriate use of vector conventions and clear flow of the required proof.

Proof: $\underset{\sim}{r} = r\cos(\omega t)\hat{i} + r\sin(\omega t)\hat{j}$

$\dot{\underset{\sim}{r}} = -r\omega\sin(\omega t)\hat{i} + r\omega\cos(\omega t)\hat{j}$

$\ddot{\underset{\sim}{r}} = -r\omega^2\cos(\omega t)\hat{i} - r\omega^2\sin(\omega t)\hat{j}$

$|\ddot{\underset{\sim}{r}}| = \underset{\sim}{a} = \sqrt{(-r\omega^2\cos(\omega t))^2 + (-r\omega^2\sin(\omega t))^2}$

$= \sqrt{(-r\omega^2)^2(\cos^2(\omega t) + \sin^2(\omega t))}$

$= \sqrt{(-r\omega^2)^2}$

$|\underset{\sim}{a}| = r\omega^2$

Now, consider the velocity $|v| = \sqrt{(r\omega\sin\omega t)^2 + (r\omega\cos\omega t)^2}$

$= \sqrt{(r\omega)^2(\sin^2\omega t + \cos^2\omega t)}$

$= r\omega$

So, $v^2 = r^2\omega^2$

$\therefore |\underset{\sim}{a}| = \frac{r^2\omega^2}{r} = r\omega^2 = \frac{v^2}{r}$

$\therefore |\underset{\sim}{a}| = \frac{v^2}{r}$ as required

Paper 2, Question 18

The following excerpt is from Question 18 of Paper 2. This complex unfamiliar question required students to determine a required probability based on given probabilities relating to a normally distributed random variable, X, and the random sampling distribution of sample mean, $\overline{X}$.

Effective student responses:

- used the given information to develop relationships between the population mean, μ, the population standard deviation, σ, and the sample size, n
- demonstrated a clear understanding of cumulative normal distribution probabilities associated with X and $\overline{X}$
- modelled clear communication of statistical information including the sample standard deviation, $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$

- used the given relevant information to develop simultaneous equations
- effectively used technology to determine the value of population parameters and the required probability.

These excerpts have been included:

- to demonstrate an effective use of the graphics calculator, in particular, in regard to the use of relevant statistical functions
- to demonstrate the effective development and solution of simultaneous equations
- to model clear communication of statistical information and notations
- to demonstrate that any alternative solutions used by students are always considered in the marking operation.

Excerpt 1

$Pr(X \le 4) = 0.5 - 0.3 = 0.2.$

$Pr(\bar{X} \le 4) = 0.5 - (0.77 - 0.3) = 0.03.$

$Pr(\bar{X} \ge 6) = 0.2.$

$Pr(X \le 4) = Pr(\bar{x} \ge 6)$ NB: $z = \frac{x-\mu}{\sigma}$ or $z = \frac{x-\mu}{\sigma/\sqrt{n}}.$

$\frac{4-\mu}{\sigma} = -\left(\frac{6-\mu}{\sigma/\sqrt{n}}\right) = \text{InvNorm}(0.2, 0, 1)$

① $\frac{4-\mu}{\sigma} = -0.8416.$ & ② $\frac{6-\mu}{\sigma/\sqrt{n}} = 0.8416$

$Pr(\bar{X} \le 4) = 0.03.$

$\frac{4-\mu}{\sigma/\sqrt{n}} = \text{InvNorm}(0.03, 0, 1)$

③ $\frac{4-\mu}{\sigma/\sqrt{n}} = -1.8808.$

① : $\frac{4-\mu}{\sigma} = -0.84$

$4-\mu = -0.84\sigma.$

③ $\frac{4-\mu}{\sigma/\sqrt{n}} = -1.8808$

$\frac{-0.84\sigma}{\sigma/\sqrt{n}} = -1.8808$

$-0.84\sqrt{n} = -1.8808$

$n = 4.99 \approx 5.$

sub $n = 5$ into ② & ③.

$\frac{6-\mu}{\sigma/\sqrt{5}} = 0.8416$ $\frac{4-\mu}{\sigma/\sqrt{5}} = -1.8808.$

$\frac{6-\mu}{\sigma} = \frac{0.8416}{\sqrt{5}} = 0.376$ $\frac{4-\mu}{\sigma} = \frac{-1.8808}{\sqrt{5}} = -0.841.$

$\frac{6-\mu}{\sigma} = 0.376$ $\frac{4-\mu}{\sigma} = -0.841$

On GDC Simult Eq solve: $\mu = 5.3817$, $\sigma = 1.6427.$

$Pr(4 \le X \le 6) = \text{Normcdf}(4, 6, 5.3817, 1.6427)$

$= 0.4465$

$\approx 0.45.$

Excerpt 2

In figure 1, the area beneath 4 = (0.5 − 0.3) = 0.2, as 30% of the distribution for X lies ~~between~~ 4 and μ.

Thus, the z-score for 4 is invNorm(0.2, 0, 1) = −0.8416

In figure 2, the area beneath 6 = (0.5 + 0.3) = 0.8, as 30% of the distribution for $\bar{x}$ lies between μ and 6. Thus the z-score is 0.8416. Let μ and σ be the parameters for X. Then $E(\bar{x}) = \mu$ and $\sigma(\bar{x}) = \frac{\sigma}{\sqrt{n}}$.

Shift the z-scores. Since $z = \frac{x-\mu}{\sigma}$, for figure 1,

① $-0.841621 = \frac{4-\mu}{\sigma}$ ⇒ ~~−0.841621~~

For figure 2 ② $0.841621 = \frac{6-\mu}{\frac{\sigma}{\sqrt{n}}}$. Moreover, the z-score for 4 in figure 2 (for $\bar{x}$) is ~0.5 − 0.47 = 0.03, as we have

The z-score is invNorm(0.03, 0, 1) = −1.88079. Thus

③ $-1.88079 = \frac{4-\mu}{\frac{\sigma}{\sqrt{n}}}$. Let $\mu = x$ and $\frac{\sigma}{\sqrt{n}} = y$.

We can lin solve using $\bar{x}$ parameters for these values with equations ② and ③

$x = \mu = 5.38171$, $y = \frac{\sigma}{\sqrt{n}} = 0.7346$ (using GDC)

Sub in ① to find σ. $-0.841621 = \frac{4-5.38171}{\sigma}$

$\sigma = 1.64172$. Then to find $P(4 \le X \le 6)$ we just use normCdf(4, 6, 5.38171, 1.64172) = 0.4468

z-score equations used:	Answer:
① $-0.841621\sigma = 4-\mu$	$P(4 \le X \le 6) = 0.4468$
② $0.841621\frac{\sigma}{\sqrt{n}} = 6-\mu$	
③ $-1.88079\frac{\sigma}{\sqrt{n}} = 4-\mu$	

Additional advice

- Teachers should ensure students are competent at performing algebraic procedures based on prior learning or assumed knowledge, including simplifying expressions (e.g. $(t^2+1)-(4t-9)=t^2-4t+10$), applications of index laws (e.g. $2\times 5^{k+1}=10\times 5^k$) and calculations based on key features of a curve (e.g. y-intercept of a line given in vector form).
- Teachers should ensure students understand the difference between inverse trigonometric functions and reciprocal trigonometric functions, e.g. $\sin^{-1}(\theta)$ and $\frac{1}{\sin(\theta)}$.
- Teachers should encourage students to become more familiar with the full capabilities of graphics calculator facilities, including determining the solution of a range of equations, determining the minimum/maximum value of a function and calculating the derivative of a function at a given point.
- Teachers should encourage students to follow the instructions in the question and response book, e.g. ‘Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.’
- Teachers should encourage students to read each question carefully and ensure that their response addresses the question posed, e.g. ‘evaluate the reasonableness of the comment’, ‘use vector calculus’ and ‘determine all possible values’.