LUI | | | | | | | | | | | School code | | | | |

School name

Given name/s

Family name

Attach your barcode ID label here

Book of books used

External assessment 2024

Question and response book

Specialist Mathematics

Paper 2 — Technology-active

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- QCAA-approved calculator **permitted**.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Clear zone — margin trimmed off after completion of assessment

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

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Section 1

Instructions

- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- Choose the best answer for Questions 1–10.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Ensure you have filled an answer bubble for each question.

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Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 50 marks.

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QUESTION 11 (4 marks)

A company claims that the mean battery life of their latest model of smartphone is 9.5 hours.

To test this claim, the battery lives of a random sample of 40 of the smartphones were measured.

A sample mean of 9.31 hours and a standard deviation of 0.52 hours were calculated from this data.

a) Determine an approximate 95% confidence interval for μ. Give your answer to at least two decimal places. *[1 mark]*

b) Determine an approximate 99% confidence interval for μ. Give your answer to at least two decimal places. *[1 mark]*

A manager comments that either confidence interval could be used to support the company's claim.

c) Use your results from Questions 11a) and 11b) to evaluate the reasonableness of the manager's comment. Justify your decision using mathematical reasoning. *[2 marks]*

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QUESTION 12 (4 marks)

A system of linear equations is given by

$$x - 2y - 2z = -6$$
$$-3x - y + z = 2$$
$$2x + 3y - 5z = 10$$

a) Express the system of equations as a matrix equation of the form $\boldsymbol{AX} = \boldsymbol{B}$, where $\boldsymbol{A}$ is a 3×3 matrix and both $\boldsymbol{X}$ and $\boldsymbol{B}$ are 3×1 column vectors. *[1 mark]*

b) Use matrix algebra to express $\boldsymbol{X}$ in terms of $\boldsymbol{A}$ and $\boldsymbol{B}$. *[1 mark]*

c) Use your result from Question 12b) to determine the solution of the system of equations. *[1 mark]*

d) Verify your result from Question 12c) using one of the given linear equations. *[1 mark]*

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QUESTION 13 (5 marks)

A drone travels vertically from point A at a constant speed of 8 m s^{-1} over time t for $t \geq 0$ seconds.

Observation of the drone is made from point B, which is 50 m horizontally from point A.

When the drone is y metres above point A, it is z metres from point B as shown.

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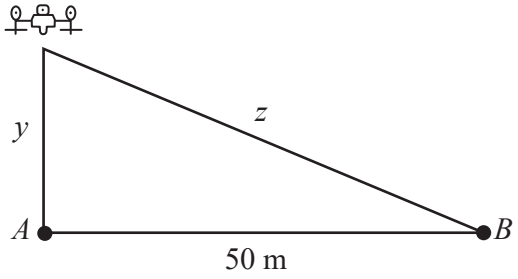

a) Determine an equation expressing z^2 in terms of y^2. *[1 mark]*

b) State the value of $\frac{dy}{dt}$. *[1 mark]*

c) Use your results from Questions 13a) and 13b) to determine the rate at which z is increasing with respect to time when the drone is 20 metres above point A. *[3 marks]*

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QUESTION 14 (5 marks)

The height of Year 12 students at a school is normally distributed, with a mean height of 168.6 cm and standard deviation of 12.7 cm.

The heights of a random sample of 20 of these students are recorded.

a) Explain why it can be assumed that the sample means for random samples of the heights of students from this school are normally distributed. *[1 mark]*

b) Determine the probability that the mean height of this sample will be greater than 170 cm. *[2 marks]*

There is a 75% probability that the mean height of this sample will lie within $\pm h$ cm of the population mean.

c) Determine $P\left(\overline{X} \geq 168.6 + h\right)$. *[1 mark]*

d) Use your result from Question 14c) to determine the value of h. *[1 mark]*

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QUESTION 15 (8 marks)

The vectors representing the position (m) of particles A and B are given by $\boldsymbol{r}_{\mathrm{A}} = (4t-9)\hat{\boldsymbol{i}} - 2(5-t)\hat{\boldsymbol{j}} - 8\hat{\boldsymbol{k}}$ and $\boldsymbol{r}_{\mathrm{B}} = (t^2+1)\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}} + (4-at^2)\hat{\boldsymbol{k}}$ respectively, where t is the time of motion for $0 \le t \le 10$ seconds.

a) Show that particle A passes through the point $P(5, -3, -8)$. *[2 marks]*

b) Given that particle B also passes through point P, determine the value of a. *[2 marks]*

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c) Determine the vector that represents the displacement of particle B relative to particle A during the given time of motion. Express your answer in simplest form. *[1 mark]*

d) Use your result from Question 15c) to determine the shortest distance between particles A and B during the given time of motion. *[3 marks]*

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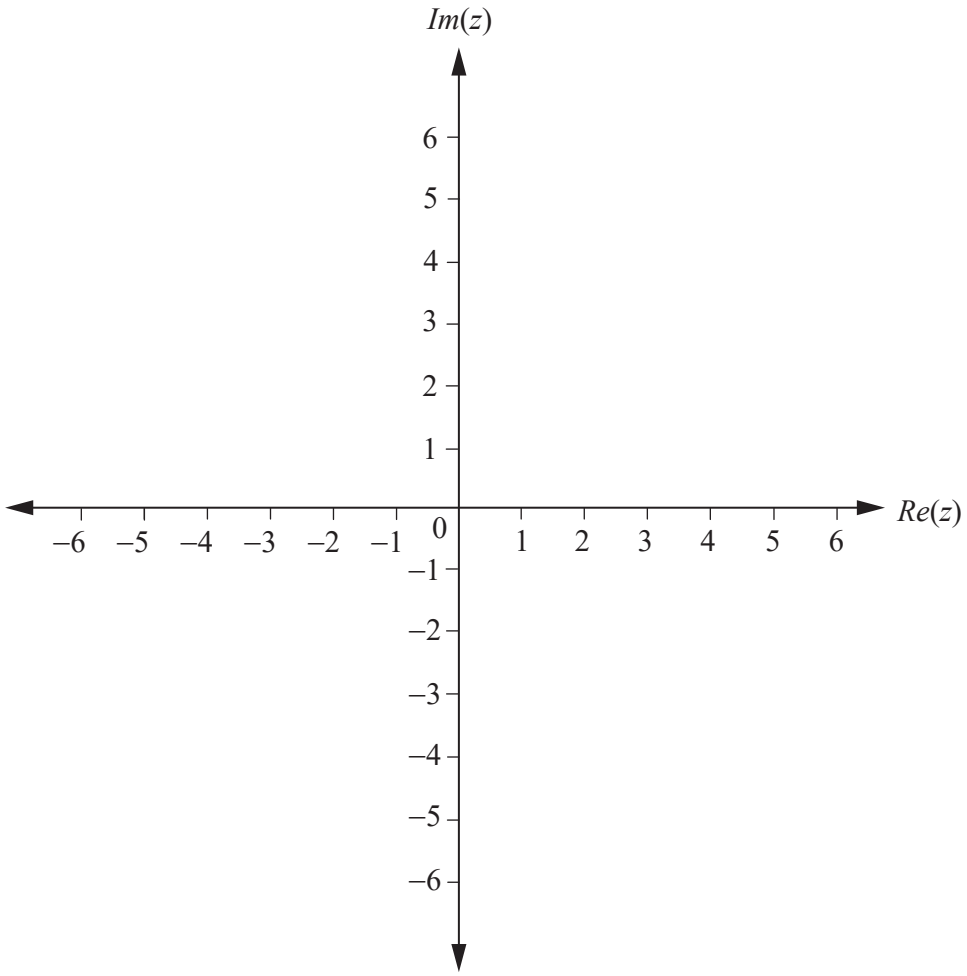
QUESTION 16 (6 marks)

Two subsets of the complex plane are $S = \{z : |z-1| = 4\}$ and $T = \left\{z : \arg(z+i) = \frac{\pi}{3}\right\}$, where $z \in C$.

Determine the complex number/s where S and T intersect. Leave your answer/s in Cartesian form.

Provide an Argand diagram with a sketch of subsets S and T as part of your solution.



Note: If you make a mistake in the diagram, cancel it by ruling a single diagonal line through your work and use the additional response space at the back of this question and response book.

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QUESTION 17 (6 marks)

An object moves with a constant speed of v in a circular path.

The position vector of the object is given by

$$\boldsymbol{r} = r\cos(\omega t)\hat{\boldsymbol{i}} + r\sin(\omega t)\hat{\boldsymbol{j}}$$

where

- r is the radius (metres) of the circle
- ω is the angular velocity (radians per second)
- t is the time (seconds) of motion for $t \geq 0$.

Use vector calculus to prove that the magnitude of the acceleration of the object is $|\boldsymbol{a}| = \dfrac{v^2}{r}$.

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QUESTION 18 (6 marks)

A random variable X is normally distributed, with a known mean μ and standard deviation σ.
In figure 1, the shaded region between 4 and μ represents 30% of the distribution of X.

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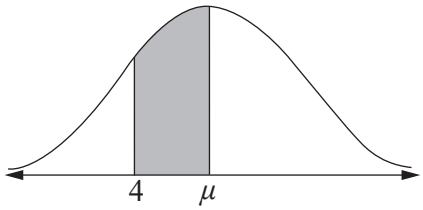

Figure 1

Consider the distribution of $\bar{X}$ based on repeated random sampling of X using a certain sample size.
In figure 2, the shaded region between μ and 6 represents 30% of the distribution of $\bar{X}$.

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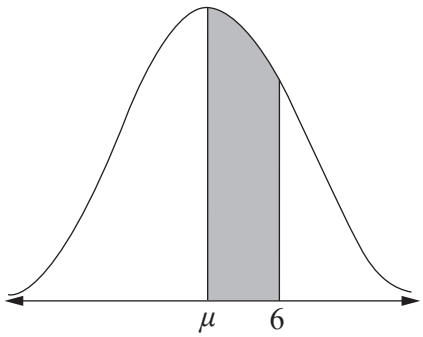

Figure 2

Given $P(4 \leq \bar{X} \leq 6) \approx 0.77$, determine $P(4 \leq X \leq 6)$.

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QUESTION 19 (6 marks)

An experiment researching the population changes of a certain species of insect was conducted over a four-week period. The insect has two distinct stages in its two-week lifespan. Each stage is approximately one week in length.

A constant proportion of females survive from stage 1 into stage 2.

The ratio of the reproduction rate for females in stage 2 to females in stage 1 is 2:1. All offspring are born into stage 1.

The number of females in each stage was measured initially and then again after two weeks as shown.

Female population	Stage 1	Stage 2
Initially	48	32
After two weeks	25	21

Use a matrix approach to estimate the total number of females after four weeks.

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ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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ADDITIONAL PAGE FOR STUDENT RESPONSES

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ADDITIONAL RESPONSE SPACE FOR QUESTION 16

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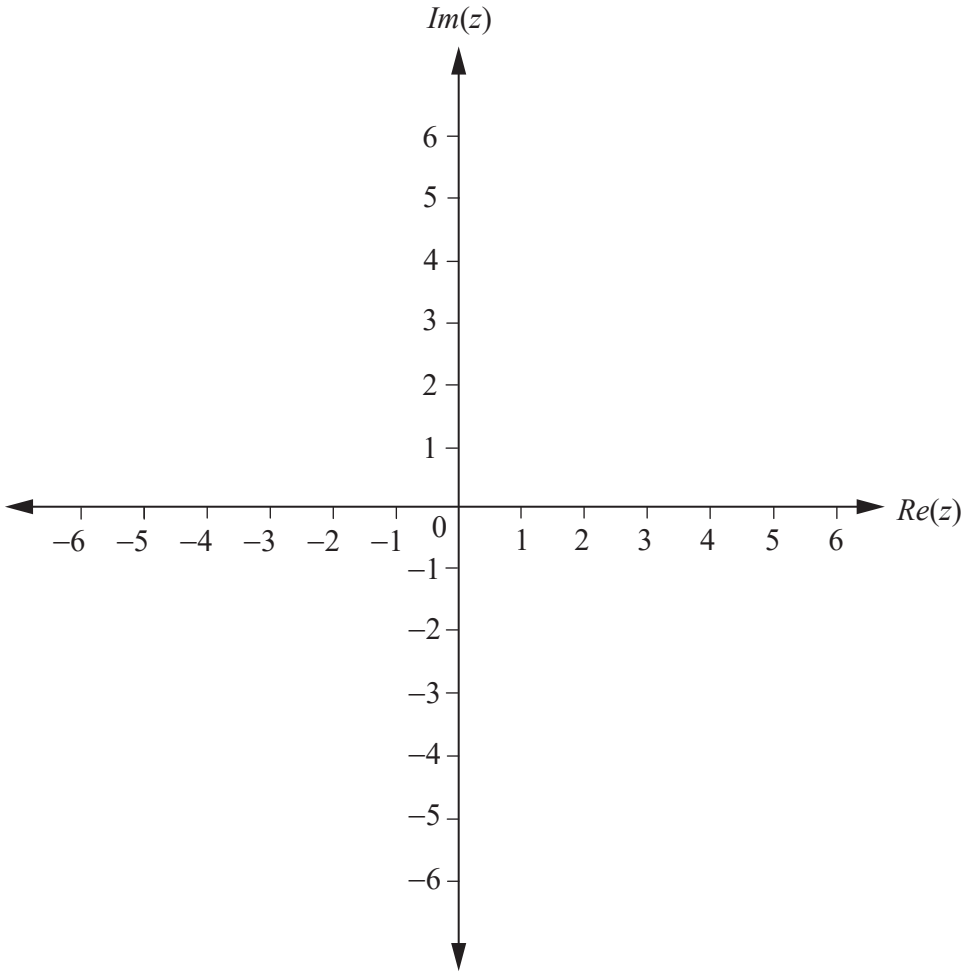

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