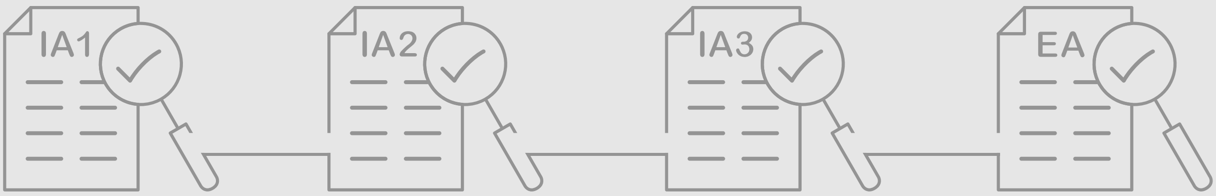
Mathematical Methods subject report

2024 cohort

January 2025



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Contents

Introduction

The annual subject reports seek to identify strengths and opportunities for improvement of internal and external assessment processes for all Queensland schools. The 2024 subject report is the culmination of the partnership between schools and the QCAA. It addresses school-based assessment design and judgments, and student responses to external assessment for General and General (Extension) subjects. In acknowledging effective practices and areas for refinement, it offers schools timely and evidence-based guidance to further develop student learning and assessment experiences for 2025.

The report also includes information about:

- how schools have applied syllabus objectives in the design and marking of internal assessments
- how syllabus objectives have been applied in the marking of external assessments
- patterns of student achievement.

The report promotes continuous improvement by:

- identifying effective practices in the design and marking of valid, accessible and reliable assessments
- recommending where and how to enhance the design and marking of valid, accessible and reliable assessment instruments
- providing examples that demonstrate best practice.

Schools are encouraged to reflect on the effective practices identified for each assessment, consider the recommendations to strengthen assessment design and explore the authentic student work samples provided.

Audience and use

This report should be read by school leaders, subject leaders, and teachers to:

- inform teaching and learning and assessment preparation
- assist in assessment design practice
- assist in making assessment decisions
- help prepare students for internal and external assessment.

The report is publicly available to promote transparency and accountability. Students, parents, community members and other education stakeholders can use it to learn about the assessment practices and outcomes for senior subjects.

Subject highlights

430
schools offered Mathematical Methods

90.61%
agreement with provisional marks for IA1

97.40%
of students received a C or higher

Subject data summary

Subject completion

The following data includes students who completed the General subject.

Note: All data is correct as at January 2025. Where percentages are provided, these are rounded to two decimal places and, therefore, may not add up to 100%.

Number of schools that offered Mathematical Methods: 430.

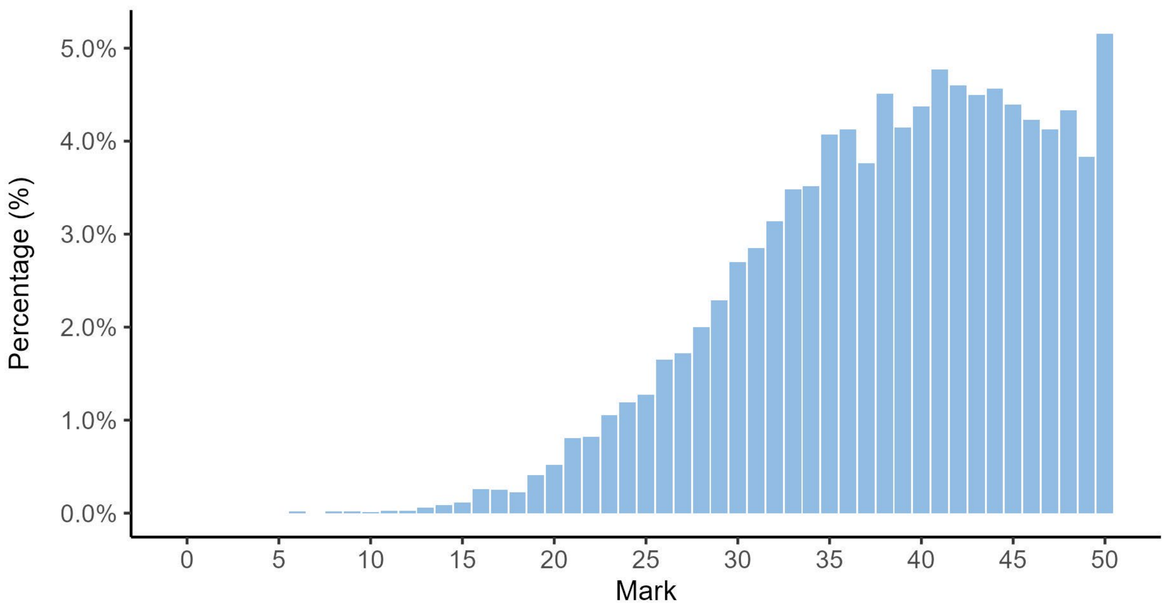
Completion of units	Unit 1	Unit 2	Units 3 and 4
Number of students completed	15,137	13,374	11,533

Units 1 and 2 results

Number of students	Satisfactory	Unsatisfactory
Unit 1	14,059	1,078
Unit 2	11,538	1,836

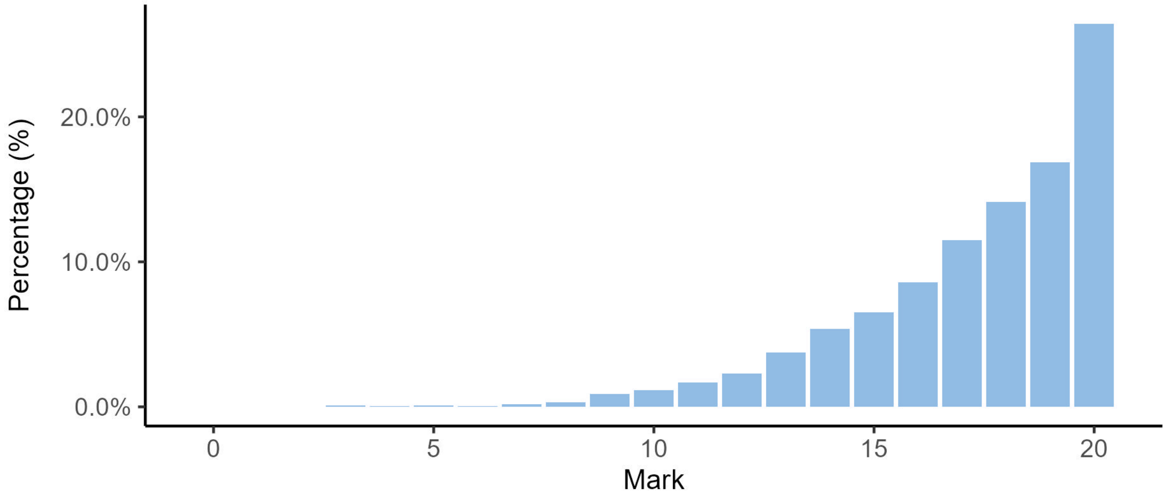
Units 3 and 4 internal assessment (IA) results

Total marks for IA

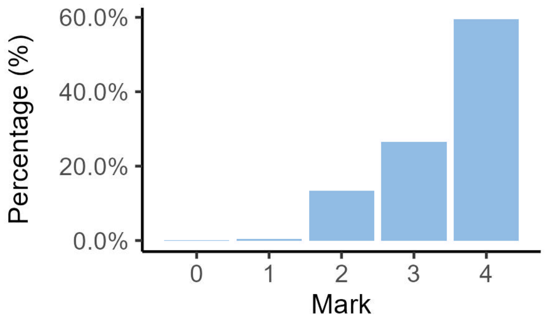


IA1 marks

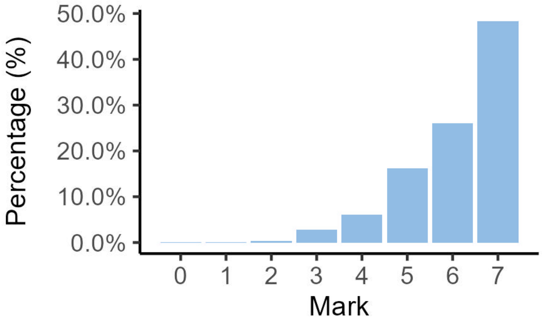
IA1 total



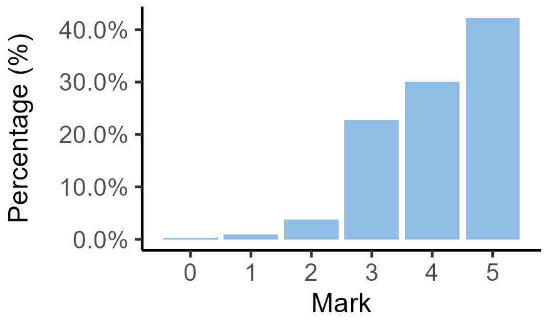
IA1 Criterion: Formulate



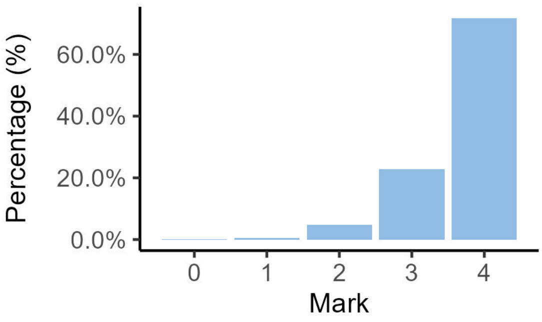
IA1 Criterion: Solve



IA1 Criterion: Evaluate and verify

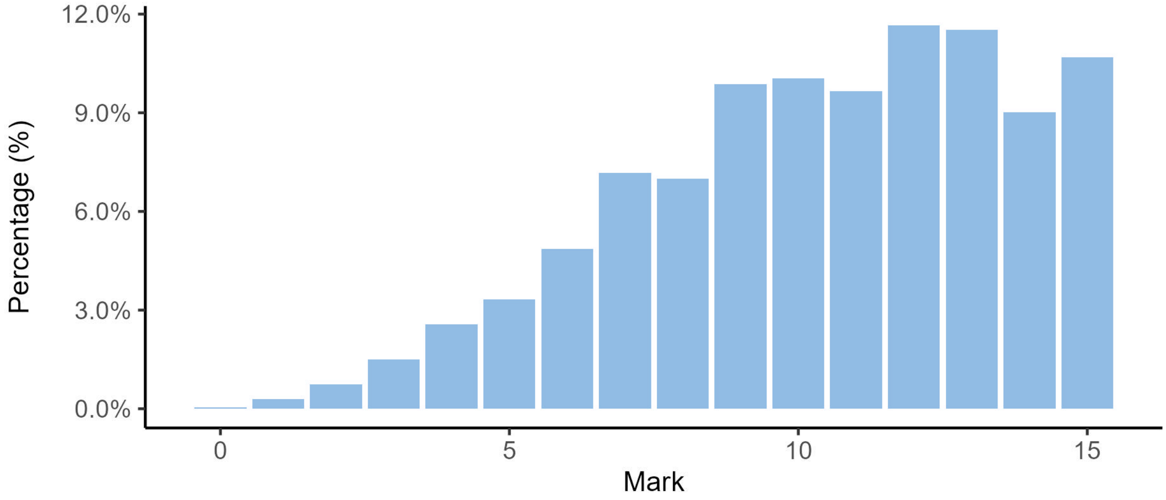


IA1 Criterion: Communicate

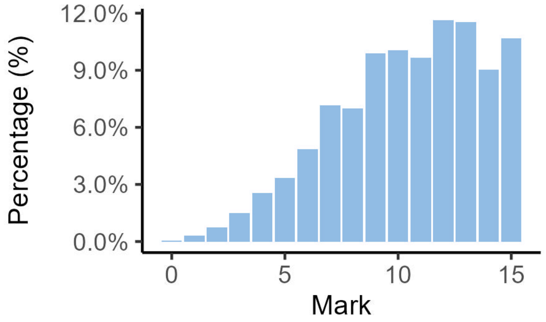


IA2 marks

IA2 total

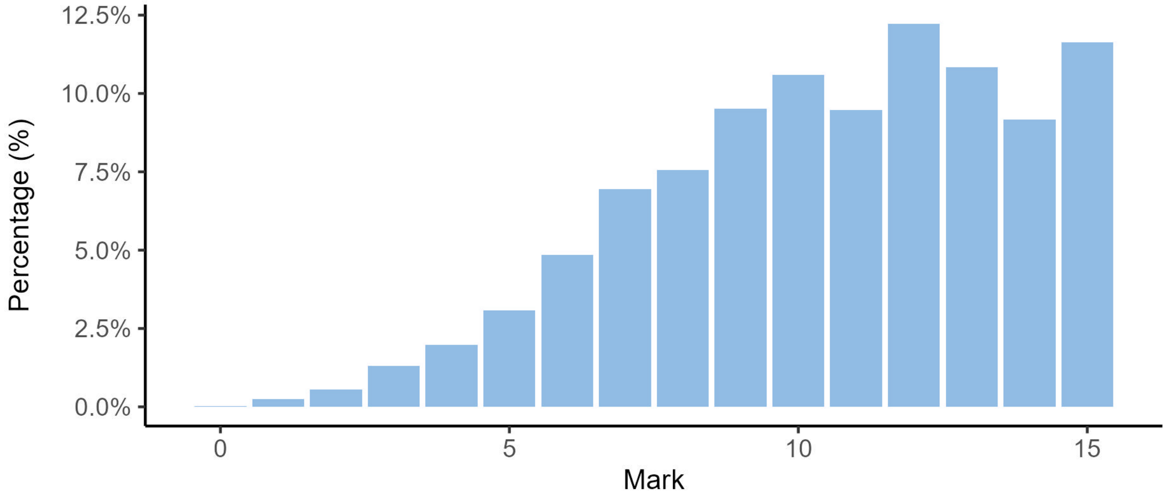


IA2 Criterion: Foundational knowledge and problem-solving

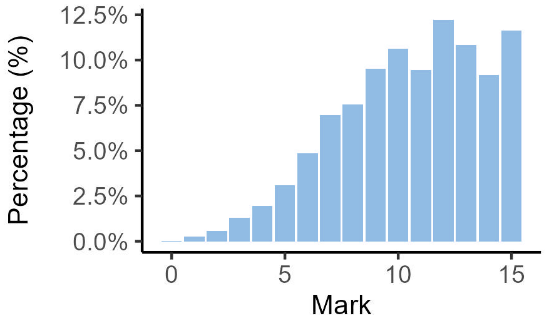


IA3 marks

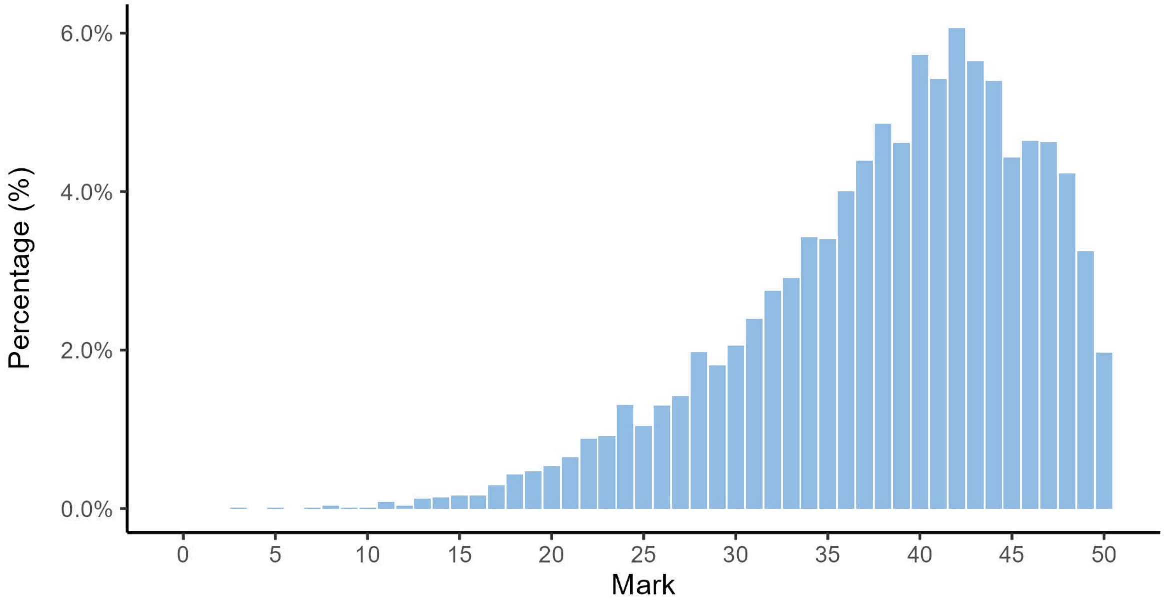
IA3 total



IA3 Criterion: Foundational knowledge and problem-solving

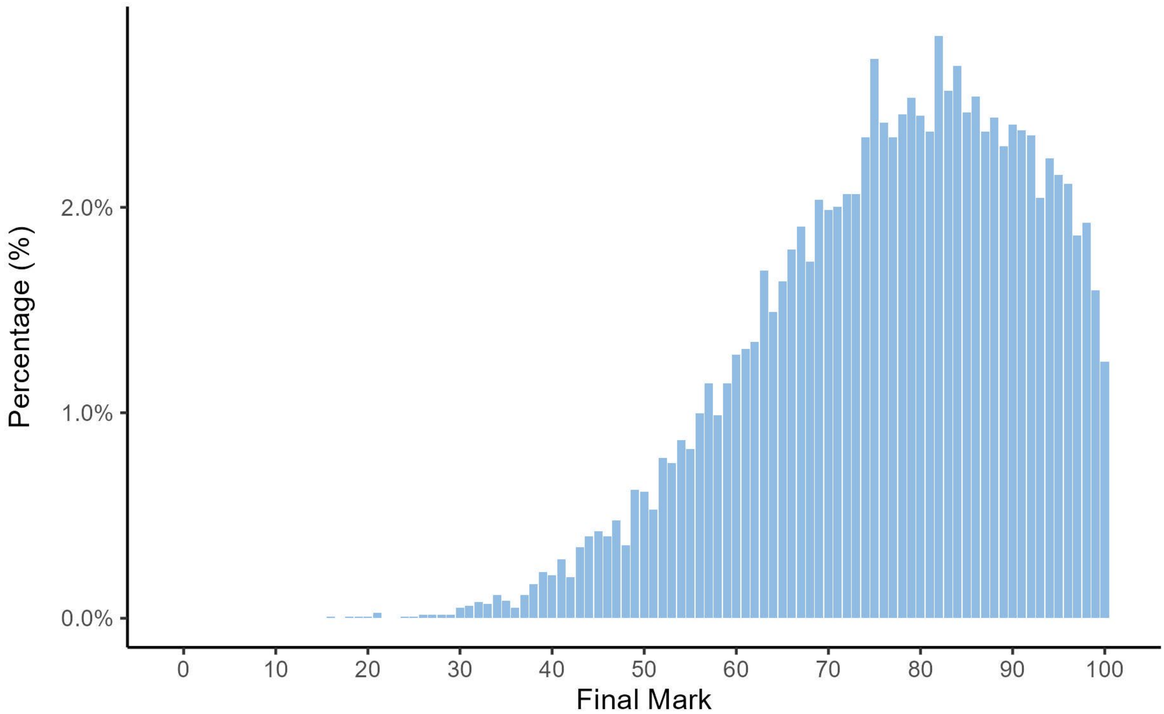


External assessment (EA) marks



Final subject results

Final marks for IA and EA



Grade boundaries

The grade boundaries are determined using a process to compare results on a numeric scale to the reporting standards.

Standard	A	B	C	D	E
Marks achieved	100–86	85–69	68–45	44–21	20–0

Distribution of standards

The number of students who achieved each standard across the state is as follows.

Standard	A	B	C	D	E
Number of students	3,686	4,650	2,897	296	4

Internal assessment

The following information and advice relate to the assessment design and assessment decisions for each IA in Units 3 and 4. These instruments have undergone quality assurance processes informed by the attributes of quality assessment (validity, accessibility and reliability).

Endorsement

Endorsement is the quality assurance process based on the attributes of validity and accessibility. These attributes are categorised further as priorities for assessment, and each priority can be further broken down into assessment practices.

Data presented in the Assessment design section identifies the reasons why IA instruments were not endorsed at Application 1, by the priority for assessment. An IA may have been identified more than once for a priority for assessment, e.g. it may have demonstrated a misalignment to both the subject matter and the assessment objective/s.

Refer to *QCE and QCIA policy and procedures handbook v6.0*, Section 9.5.

Percentage of instruments endorsed in Application 1

Instruments submitted	IA1	IA2	IA3
Total number of instruments	430	430	424
Percentage endorsed in Application 1	74	34	40

Confirmation

Confirmation is the quality assurance process based on the attribute of reliability. The QCAA uses provisional criterion marks determined by teachers to identify the samples of student responses that schools are required to submit for confirmation.

Confirmation samples are representative of the school's decisions about the quality of student work in relation to the instrument-specific marking guide (ISMG), and are used to make decisions about the cohort's results.

Refer to *QCE and QCIA policy and procedures handbook v6.0*, Section 9.6.

The following table includes the percentage agreement between the provisional marks and confirmed marks by assessment instrument. The Assessment decisions section of this report for each assessment instrument identifies the agreement trends between provisional and confirmed marks by criterion.

Number of samples reviewed and percentage agreement

IA	Number of schools	Number of samples requested	Number of additional samples requested	Percentage agreement with provisional marks
1	425	3,266	11	90.61
2	425	3,268	0	100.00
3	425	3,259	0	100.00

Internal assessment 1 (IA1)

Problem-solving and modelling task (20%)

This assessment focuses on the interpretation, analysis and evaluation of ideas and information. It is an independent task responding to a particular situation or stimuli. While students may undertake some research in the writing of the problem-solving and modelling task, it is not the focus of this technique. This assessment occurs over an extended and defined period of time. Students will use class time and their own time to develop a response.

The problem-solving and modelling task must use subject matter from one or both of the following topics in Unit 3:

- Topic 2: Further differentiation and applications 2
- Topic 3: Integrals.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions
Alignment	50
Authentication	18
Authenticity	12
Item construction	22
Scope and scale	17

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- provided authentic opportunities for students to solve mathematical problems within a real-world context, e.g. determining the area of a country, designing a waterslide or rollercoaster, modelling the cooling of a cup of coffee or modelling the growth of a population
- required students to use technology beyond simple computation and word processing, such as the use of graphing applications to develop appropriate models
- featured task requirements of appropriate scope and scale to allow students to address all stages of the problem-solving and modelling approach.

Practices to strengthen

It is recommended that assessment instruments:

- specify the types of functions required for tasks assessing Unit 3 Topic 2 to avoid students solving the problem using solely Unit 2 subject matter, e.g. 'Use exponential, logarithmic and trigonometric functions to …' or 'Three of your chosen functions must be functions studied in Unit 3'

- avoid scaffolding or task instructions that direct students on how to formulate and solve the problem (e.g. avoid 'Use integration to …') so students can demonstrate their knowledge and understanding of the criteria
- include well-described checkpoints and authentication strategies that comply with the QCAA drafting policy and academic integrity guidelines, including a checkpoint that indicates the submission of one complete or near-complete draft for feedback (*QCE and QCIA policy and procedures handbook v6.0*, Section 8.2.5)
- require at most only limited research, as research is not the focus of this assessment technique.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions
Bias avoidance	3
Language	17
Layout	1
Transparency	12

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- provided clear instructions to students about the requirements of the task
- provided a clear layout, where text and images appeared aligned and appropriately scaled relative to the size of the page.

Practices to strengthen

It is recommended that assessment instruments:

- are reviewed before submission to check the text for typographical, grammatical, punctuation and spelling errors.

Additional advice

- In the scaffolding section of the instrument, schools can refer to the approach to problem-solving and mathematical modelling from the syllabus (Syllabus section 1.2.4, Figure 4), or may develop a task-specific flowchart. Where a school chooses to use a task-specific flowchart, it should match the current instrument's context and task.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Formulate	92.94	6.12	0.94	0
2	Solve	97.65	1.88	0.47	0
3	Evaluate and verify	96.71	3.06	0.24	0
4	Communicate	98.59	0.94	0.47	0

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- judgments made about the accurate use of complex procedures were supported by clear evidence in a student's application of subject matter to reach a solution to the task. Complexity was recognised in student work that developed an approach to solve the task using multiple steps drawn from Unit 3 Topics 2 and/or 3 syllabus descriptors
- the accurate and appropriate use of technology was identified when students used technology that was chosen to support the process of solving the problem. The appropriate use of technology went beyond the simple inclusion of word processing and graphing, and made use of the technology to help solve the problem
- judgments made about the communication of a response were supported by the correct use of language that accurately applied technical vocabulary and procedural vocabulary. Further, responses with a clear introduction that was relevant to the task, and a conclusion that responded to the problem that had been solved, effectively demonstrated characteristics of the Communicate criterion.

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- when making judgments for the Formulate criterion, evidence for the documentation of assumptions and observations should be relevant to the solving of the problem and go beyond restating task specifications. Appropriate documentation could identify why the assumption or observation would be relevant to the task and identify their effects on the model used in the task
- judgments made about the evaluation of the reasonableness of solutions in the Evaluate and verify criterion need to consider more than the results. Evidence for this could also consider the observations and the assumptions, which may be demonstrated by clearly identifying the relevance and effects these would have on the solution obtained using the model

- evidence for the justification of decisions should show use of mathematical reasoning in the student's response to support why decisions are right or reasonable.

Samples

The following excerpt demonstrates documentation of assumptions and observations.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

Observations have been made through the Widmark formula and the variables within. It has been observed that all variables are proportional to each other, as it is a logarithmic function. For example, BAC will increase if the amount of alcohol consumed also increases

accurate documentation of relevant observations.

(Searle, 2014). It has also been observed that an empty stomach absorbs alcohol at a rate of 6 compared to a stomach with food, which is at a rate of 2.3. The Widmark factor also showed that men have an $r \approx 0.7$ and women have an $r \approx 0.6$. Thus, the following assumptions have been made:

- Only one variable will be changed in each scenario. This is to ensure that the comparison is accurate, and to identify clearer how the changed variables affect BAC.
- Alcohol consumption is the controlled variable and stays the same for all scenarios.
- Height and mass are the same for a scenario to ensure that change is minimal.
- *t* is the independent variable and thus will be represented as x, while *B* is the dependent variable and thus will be represented as y or $f(x)$. This is so that the Widmark formula can be used to graph the logarithmic function.
- People in the scenarios take no medications or have underlying conditions that would affect BAC.
- People in scenarios are all from Australia for accurate and fair comparisons of BAC.
- The people in the scenarios are chronic drinkers—someone who drinks more than four standard drinks a day, regularly— and drinks 70g of alcohol in one sitting *(National Institute on Alcohol Abuse and Alcoholism, 2020)*.

documentation of appropriate assumptions.

The following excerpt has been included to demonstrate the evaluation of the reasonableness of a solution by also considering some assumptions and observations.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

It was assumed that the if any function modelled an acceleration greater than 9.8m/s during freefall it was unreasonable. All models stayed within this parameter.

An observation was that any models displaying a g-force higher than 6G were not reasonable as the booster cannot handle this amount of force. Function #5 has a g-force of 8.3G, a limitation to the model.

Limitations

Commented [MP14]: EVALUATE and VERIFY
- evaluation of the reasonableness of solutions by considering the results, assumptions and observations

The following excerpt demonstrates an alternative student response showing evidence of evaluating the reasonableness and validity of their solution. Initial assumptions and observations have been referenced to evaluate the reasonableness of the model and demonstrate that a valid solution was obtained.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

This model is reasonable and valid for the following reasons. Firstly, the model has a range of [0, 12] and a range of [0, ≈100] which satisfies the above assumptions. Secondly, y does not equal a negative number (as stated in the observations above) therefore making this model a valid solution. Furthermore, when the difference in gradient is calculated at each intersection, these figures satisfy the assumption made that a smooth intersection will be considered when the difference i less than 0.1.

Additional advice

- The problem-solving and mathematical modelling approach (Syllabus section 1.2.4, Figure 4) outlines good practice when developing a model. Students should use this approach to aid them in their development of a solution, which could provide evidence for the elements of the ISMG.
- Teacher annotations on assessment responses would help to identify the evidence teachers used to determine where students have met descriptors from the ISMG.
- In a student response that exceeds the IA1 conditions outlined in the syllabus (maximum of 2000 words; up to 10 pages), teachers should annotate the student response, ISMG or instrument-specific standards to indicate which evidence was used to make a judgment. For quality assurance processes, the school must make it clear on the sample which strategy was applied, so that confirmers can review only the work that the teacher has marked. (See *QCE and QCIA policy and procedures handbook v6.0*, Section 8.2.6: Managing response length.)

Internal assessment 2 (IA2)

Examination (15%)

This examination assesses the application of a range of cognitions to a number of items, drawn from all Unit 3 topics. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions
Alignment	253
Authentication	0
Authenticity	1
Item construction	28
Scope and scale	32

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- featured a balance of technology-free and technology-active questions by either
  - constructing the instrument with separate technology-free and technology-active sections
  - providing appropriate cues, if necessary, to indicate where an algebraic response is required in technology-active questions, e.g. ‘Use algebraic procedures to …’
- included an appropriate number of questions so students could respond to all items within the available time
- provided a correct marking scheme that indicated clearly how marks were allocated within expected responses, including the allocation of a mark/s for demonstration of Assessment objective 4: evaluate the reasonableness of solutions.

Practices to strengthen

It is recommended that assessment instruments:

- representatively sample subject matter across all Unit 3 topics using questions of suitable scope and scale
- require students to demonstrate knowledge of Unit 3 subject matter and do not require any Unit 4 subject matter, nor solely assess any assumed knowledge, e.g.

- calculating the derivative of $f(x) = \frac{2x-1}{x^2-x}$ is not an appropriate question as this is assumed knowledge from Unit 2 with both the numerator and denominator of $f(x)$ being polynomials
- expecting students to use acceleration as the second derivative of displacement is not reasonable in a question as this is Unit 4 subject matter

- align complex unfamiliar questions to the degree of difficulty specification (Syllabus section 4.6.2) so
  - relationships and interactions have a number of elements but are not scaffolded, e.g. not providing a series of parts that step students through a problem
  - all the information to solve the problem is not immediately identifiable, i.e. the required procedure is not clear from the way the problem is posed and it is presented in a context with which students have had limited prior experience
- ensure that marks allocated to each question are appropriate for the complexity of skills being assessed, e.g. 'Determine $\frac{d}{dx}(4\sin(3x+2))$ at $x = 1$' could be performed in one calculator entry using technology, so allocating marks for algebraic working in a technology-active paper would not be appropriate.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions
Bias avoidance	4
Language	46
Layout	11
Transparency	25

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- featured a clear layout, with appropriately sized images, so questions appeared in their entirety on one page
- were reviewed using the **Print preview** button to ensure the examination appropriately displayed all questions and images
- used consistent fonts and formatting throughout.

Practices to strengthen

It is recommended that assessment instruments:

- use correct language conventions, and are free from spelling, grammatical and typographical errors
- are reviewed before submission to check for spelling, grammatical and typographical errors
- use the language of the assessment objectives, where applicable, e.g. 'Evaluate the reasonableness of your solution'.

Additional advice

- Schools should ensure that examinations are constructed so that the amount of reading does not disadvantage students. Multiple questions that take a long time for a student to read and interpret can limit their ability to demonstrate the knowledge and understanding of the subject matter in the allocated timeframe.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	100	0	0	0

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- marking schemes were free from errors, clearly identified the allocation of marks, and were consistently and accurately applied to all student responses
- the total marks awarded for each question were clearly annotated and the awarded total could be evidenced by the corresponding ticks
- an accurate percentage calculation was annotated on the student work and the correct percentage cut-off was used to determine a student's provisional ISMG mark, e.g. 40/60 = 66.67%, which is greater than 60% and awarded an ISMG mark of 10.

Practices to strengthen

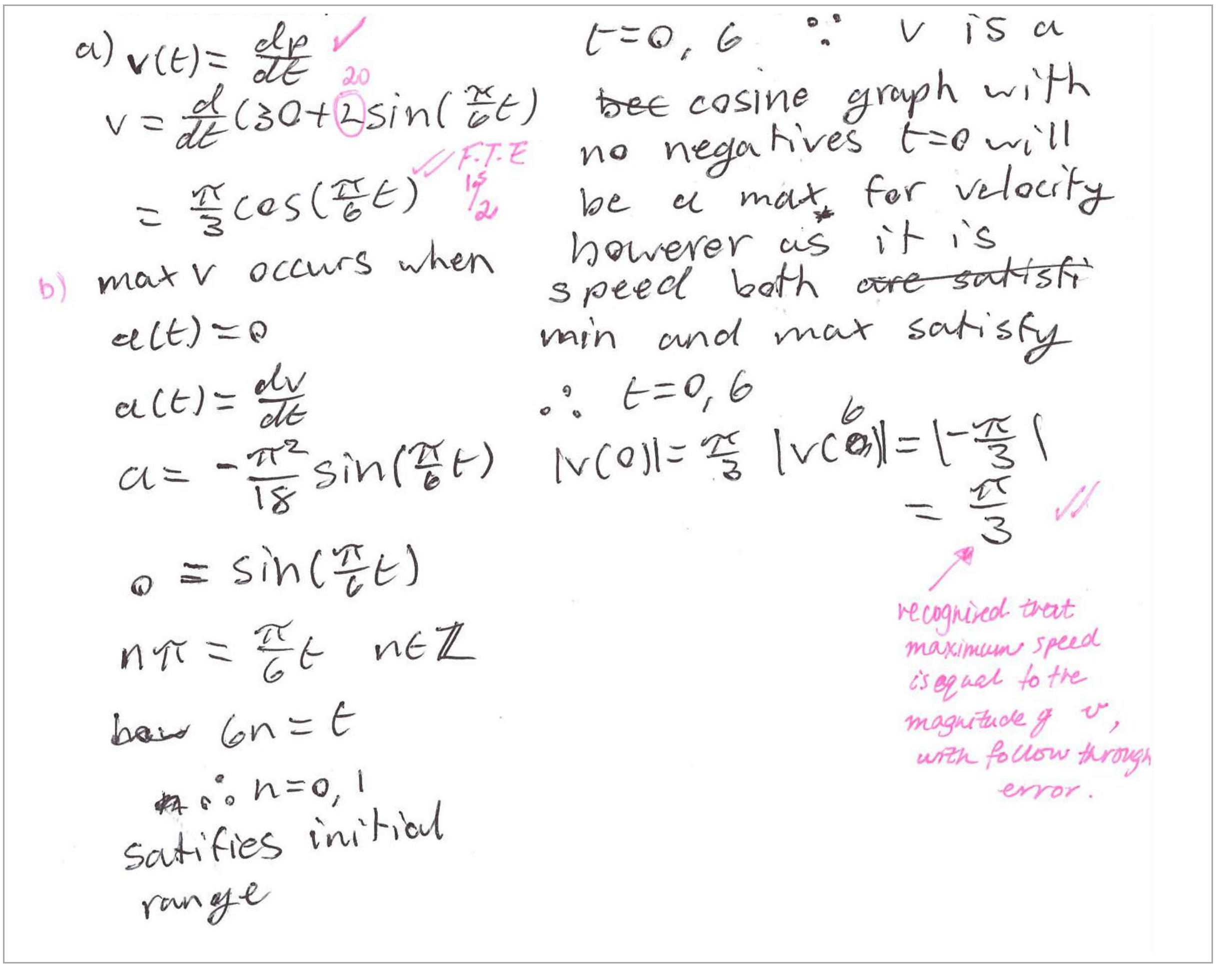
To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- when marking responses where an alternative method to the marking scheme has been used, the student work should be annotated to clearly identify where marks were awarded
- the most up-to-date version of the marking scheme is uploaded at confirmation
- comparable tasks are uploaded with the correct marking scheme at confirmation.

Samples

The following excerpt has been included because it demonstrates an alternative method that has been annotated to show how marks have been awarded. A second derivative test was expected in the school's marking scheme to identify maximum speed, but this response involved graph magnitudes and absolute values.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.


a) $v(t) = \frac{dp}{dt}$ ✓

$v = \frac{d}{dt}(30 + 2\sin(\frac{\pi}{6}t))$ 20

$= \frac{\pi}{3}\cos(\frac{\pi}{6}t)$ ✓✓ F.T.E $1\frac{1}{2}/2$

b) max v occurs when

$a(t) = 0$

$a(t) = \frac{dv}{dt}$

$a = -\frac{\pi^2}{18}\sin(\frac{\pi}{6}t)$

$0 = \sin(\frac{\pi}{6}t)$

$n\pi = \frac{\pi}{6}t \quad n \in \mathbb{Z}$

~~how~~ $6n = t$

$\therefore n = 0, 1$ satifies initial range

$t = 0, 6 \therefore$ v is a ~~bee~~ cosine graph with no negatives t=0 will be a max for velocity however as it is speed both ~~are satisfi~~ min and max satisfy

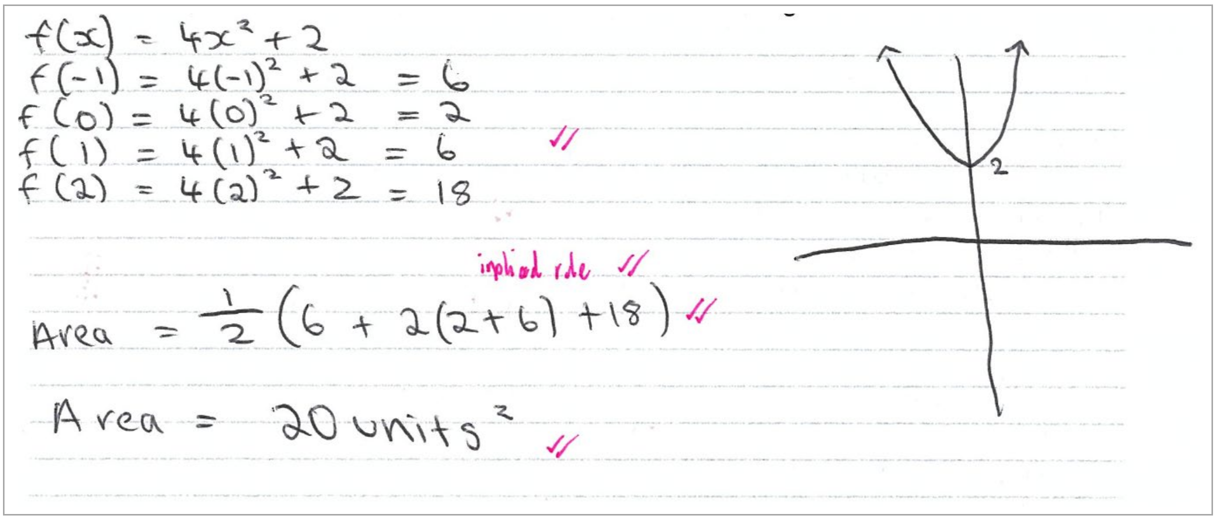
$\therefore t = 0, 6$

$|v(0)| = \frac{\pi}{3} \quad |v(6)| = |-\frac{\pi}{3}| = \frac{\pi}{3}$ ✓✓

recognised that maximum speed is equal to the magnitude of v, with follow through error.

The following excerpt demonstrates annotations that clearly indicate the awarding of an implied mark (for a mark allocated in the marking scheme for stating the trapezoidal rule).

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.


$f(x) = 4x^2 + 2$

$f(-1) = 4(-1)^2 + 2 = 6$

$f(0) = 4(0)^2 + 2 = 2$

$f(1) = 4(1)^2 + 2 = 6$ ✓✓

$f(2) = 4(2)^2 + 2 = 18$

implied rule ✓✓

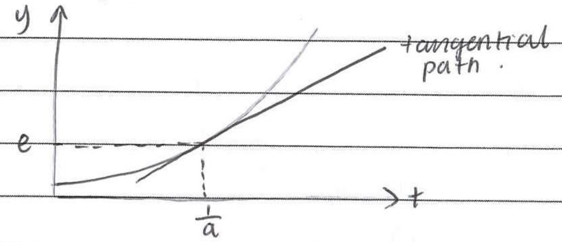
$\text{Area} = \frac{1}{2}(6 + 2(2+6) + 18)$ ✓✓

Area = 20 units^2 ✓✓

The following excerpt has been included to demonstrate the annotation and awarding of a mark to Assessment objective 4: evaluate the reasonableness of solutions.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

Evaluating for reasonableness.



$y = e^{at}$.

$y' = ae^{at}$

great

$t = \frac{1}{a} \Rightarrow y' = ae^{a(\frac{1}{a})} = ae^1 = ae$.

$\therefore$ The gradient at $t = \frac{1}{a}$ is ae.

$y - y_1 = m(x - x_1)$.

$y - e = ae(x - \frac{1}{a})$.

$y = aex - \frac{ae}{a} + e$.

$= aex - e + e = aex$.

$y = ae\,x$.

$\therefore$ Because the particle is travelling tangent to the original path, the path will be linear. When the path is linear, the velocity (gradient) is constant $\Rightarrow$ when $t = a$, velocity is ae km/h

$y' = ae$. ← no t variable.

✓✓

Additional advice

- When an emergent situation occurs, such as an error identified in the endorsed instrument either at implementation or after the administering of the instrument, it is important to contact the Endorsement team (phone 3864 0375 or email endorsement@qcaa.qld.edu.au) to discuss options that will maintain the instrument integrity. (See *QCE and QCIA policy and procedures handbook v6.0*, Section 9.5.6: Amending an endorsed assessment instrument (whole cohort).)
- Schools should apply internal quality assurance processes to check the raw mark totals and the correct application of percentage cut-offs to award the ISMG mark out of 15.

Internal assessment 3 (IA3)

Examination (15%)

This examination assesses the application of a range of cognitions to a number of items, drawn from Unit 4 Topics 1–5. Student responses must be completed individually, under supervised conditions, and in a set timeframe.

Assessment design

Validity

Validity in assessment design considers the extent to which an assessment item accurately measures what it is intended to measure and that the evidence of student learning collected from an assessment can be legitimately used for the purpose specified in the syllabus.

Reasons for non-endorsement by priority of assessment

Validity priority	Number of times priority was identified in decisions
Alignment	218
Authentication	0
Authenticity	1
Item construction	14
Scope and scale	42

Effective practices

Validity priorities were effectively demonstrated in assessment instruments that:

- were constructed with a balance of technology-free and technology-active questions
- provided simple familiar questions where the required procedure was clear from the way the problem was posed
- provided a correct marking scheme that indicated clearly how marks were allocated within expected responses
- featured an appropriate number of questions that matched the degree of difficulty specifications in the syllabus and were of a suitable scope and scale to allow students to respond to subject matter from all topics in Unit 4 within the time conditions.

Practices to strengthen

It is recommended that assessment instruments:

- provide opportunities for students to respond to Unit 4 Topic 1 subject matter by ensuring that marks are awarded for the use of the second derivative test to identify maximums and minimums when solving optimisation problems
- align complex unfamiliar opportunities to the degree of difficulty specification (Syllabus section 5.8.1) so
  - relationships and interactions have a number of elements

  - all the information to solve the problem is not immediately identifiable by avoiding providing scaffolding and/or explicit instructions to students about how to solve the problem
- provide opportunities for students to respond to Assessment objective 4: evaluate the reasonableness of solutions, including allocating a mark/s for this objective in the marking scheme
- include questions that focus on Unit 4 subject matter. Questions that can be solved using only subject matter from Units 1, 2 or 3 are not suitable, e.g. finding the displacement function from a given acceleration function is Unit 3 subject matter, whereas finding the acceleration function from a given displacement is Unit 4 subject matter.

Accessibility

Accessibility in assessment design ensures that no student or group of students is disadvantaged in their capacity to access an assessment.

Reasons for non-endorsement by priority of assessment

Accessibility priority	Number of times priority was identified in decisions
Bias avoidance	2
Language	48
Layout	5
Transparency	25

Effective practices

Accessibility priorities were effectively demonstrated in assessment instruments that:

- clearly identified the use of technology or an algebraic approach for questions where they were not separated into technology-free and technology-active papers
- used consistent fonts and formatting, and provided sufficient space for student responses
- were reviewed using the **Print preview** button to ensure the instrument displayed questions, images and page breaks appropriately.

Practices to strengthen

It is recommended that assessment instruments:

- are reviewed before submission to check they use correct language conventions and are free from spelling, grammatical and typographical errors.

Additional advice

- Ensure that instruments are constructed in such a way that the amount of reading is not disadvantageous to students. Multiple questions that take a long time for the student to read and interpret can limit their ability to demonstrate the knowledge and understanding of the subject matter in the allocated timeframe.

Assessment decisions

Reliability

Reliability is a judgment about the measurements of assessment. It refers to the extent to which the results of assessments are consistent, replicable and free from error.

Agreement trends between provisional and confirmed marks

Criterion number	Criterion name	Percentage agreement with provisional	Percentage less than provisional	Percentage greater than provisional	Percentage both less and greater than provisional
1	Foundational knowledge and problem-solving	100	0	0	0

Effective practices

Accuracy and consistency of the application of the ISMG for this IA was most effective when:

- accurate marking schemes were submitted for confirmation
- teacher annotations, when used on student responses, were clear and indicated the evidence teachers used to award marks and, where appropriate, how follow-through marks were awarded.

Practices to strengthen

To further ensure accuracy and consistency of the application of the ISMG for this IA, it is recommended that:

- internal quality assurance processes are applied to check marking decisions, raw mark totals and the correct application of percentage cut-offs to award the ISMG mark out of 15
- the correctly totalled marks and student's achievement percentage are clearly shown on the ISMG (*QCE and QCIA policy and procedures handbook v6.0,* Section 9.6.1)
- if a comparable assessment instrument is administered to a sampled student, then the school must indicate this in Student Management on the individual student's learning account and in the Confirmation application (app). Comparable assessments should be developed in the Endorsement app to ensure the correct examination and its matching marking scheme are available for the confirmation review (*QCE and QCIA policy and procedures handbook v6.0*, Section 7.4). For further information, see the quick-step guide *Upload samples* in the Help section of the Confirmation app.

Samples

The following excerpt demonstrates the appropriate application of the ISMG, where the raw mark total is shown, and the percentage cut-off is correctly applied. This example demonstrates a borderline awarded mark total and the correct interpretation of the 'greater than' feature of the ISMG.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

Instrument-specific marking guide (IA3): Examination (15%)

Criterion: Foundational knowledge and problem-solving

Assessment objectives

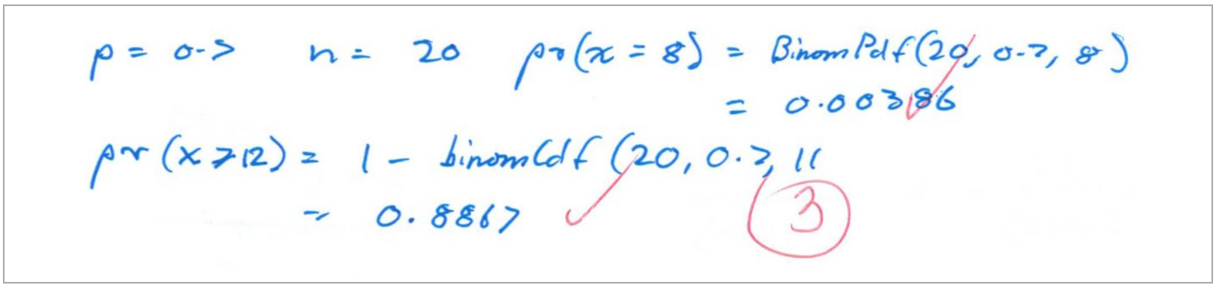
1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.

23½/50 47%

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6

The following excerpt has been included as an example of clear communication of reasoning when technology has been used to solve a multi-mark question.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.



The following excerpt demonstrates an example of how a student has communicated the use of technology to solve a cubic equation as an exact value.

Note: The characteristic/s identified may not be the only time the characteristic/s occurred throughout a response.

$$= a(x-1)(x-1)$$

$$= a(x^2 - x - x + 1)$$

$$= a(x^2 - 2x + 1)$$

$$= ax^2 - 2ax + a \checkmark$$

$$1 = \int_0^4 ax^2 - 2ax + a \, dx$$

$$1 = \left[\frac{ax^3}{3} - ax^2 + ax\right]_0^4 \quad \text{use GC - solve h}$$

$$1 = \frac{a(4)^3}{3} - a(4)^2 + 4a - (0) \quad \text{- solve h}$$

$$a = \frac{3}{28} \checkmark$$

Additional advice

- Schools are responsible for ensuring the quality, accuracy and accessibility of the required files for confirmation (*QCE and QCIA policy and procedures handbook v6.0*, Section 9.6.3). The *Confirmation submission information* for Mathematical Methods is available under Resources in the Syllabuses app in the QCAA Portal. Before submitting responses for confirmation, schools are advised to check that all scanning of student work has been completed without error. This includes ensuring that
  - no pages are missing from the response
  - all pages are visible and easy to read

  - the submitted response matches the student selected.

- Marking schemes should provide a sample response for each item and the intended mark allocation, which should be consistently adhered to in marking.

External assessment

External assessment (EA) is developed and marked by the QCAA. The external assessment for a subject is common to all schools and administered under the same conditions, at the same time, on the same day.

Examination — short response (50%)

Assessment design

The assessment instrument was designed using the specifications, conditions and assessment objectives described in the summative external assessment section of the syllabus.
The examination consisted of two papers:

- Paper 1, Section 1 consisted of multiple choice questions (10 marks)
- Paper 1, Section 2 consisted of short response questions (45 marks)
- Paper 2, Section 1 consisted of multiple choice questions (10 marks)
- Paper 2, Section 2 consisted of short response questions (45 marks).

Assessment decisions

Assessment decisions are made by markers by matching student responses to the external assessment marking guide (EAMG). The external assessment papers and the EAMG are published in the year after they are administered.

Multiple choice question responses

There were 10 multiple choice questions in Paper 1.

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Question	A	B	C	D
1	4.08	0.92	1.60	**93.25**
2	**91.95**	3.82	2.92	1.16
3	2.96	**66.31**	7.83	22.62
4	**68.18**	10.78	11.16	9.68
5	16.48	12.05	**57.89**	13.15
6	1.01	13.14	10.05	**75.62**
7	20.49	**63.28**	14.13	1.36
8	21.88	10.53	**53.34**	14.00
9	5.46	**59.02**	25.50	9.52
10	50.49	8.99	**35.35**	4.86

There were 10 multiple choice questions in Paper 2.

Percentage of student responses to each option

Note:

- The correct answer is **bold** and in a blue shaded table cell.
- Some students may not have responded to every question.

Question	A	B	C	D
1	1.88	2.22	**94.37**	1.24
2	**62.21**	9.02	25.92	2.23
3	10.64	3.71	**74.24**	10.82
4	6.14	**71.20**	9.83	11.91
5	7.36	7.54	6.72	**77.57**
6	**81.06**	7.66	8.47	2.22
7	3.57	**70.11**	22.91	2.61
8	62.25	5.52	4.23	**27.32**
9	12.51	12.22	**65.15**	9.25
10	4.69	**83.29**	5.41	5.71

Effective practices

Overall, students responded well to:

- opportunities to use the formulas of differential and integral calculus, e.g. in questions that required determining formulas for the first and second derivative or those involving the integration of exponential or trigonometric functions
- using the approximate confidence interval relationship for estimating population proportion, including the margin of error
- recognising qualitative features of a logarithmic graph, such as the vertical asymptote and horizontal translation, and solving a logarithmic function without technology
- using technology in a range of contexts, such as equation-solving applications, statistical applications, and evaluating derivatives or rates of change
- opportunities to interpret and solve problems that involved exponential and logarithmic functions and their derivatives.

Practices to strengthen

When preparing students for external assessment, it is recommended that teachers consider:

- providing technology-free opportunities for students to develop and maintain their algebraic and arithmetic skills in procedures, such as manipulating equations, expanding brackets, simplifying algebraic and fractional expressions, working with powers and interchanging between fractions and decimals. In Paper 1, Question 13b), many students were unable to progress to the correct integration of the function due to errors made in expanding the given squared bracket and simplifying the resulting terms. In Paper 1, Question 15b), successful responses often demonstrated the interchange between fractions and decimals to assist with the calculation

- increasing students' opportunities to engage in two- or three-dimensional problems in technology-free situations. In particular, in problem-solving situations where students are required to use estimation of calculated values. Further, this could improve students' ability to discern the relationships between the angles, lengths, areas or volumes involved in physical contexts. In Paper 1, Question 18, many students could progress part of the way into the problem but were unable to reach a feasible outcome, often due to not being able to convert the progressing calculation to a reasonable approximation
- providing regular opportunities to engage with problems that require the use of the second derivative across a variety of contexts. In Paper 1, Question 19, a model for the acceleration needed to be developed. In Paper 2, Question 18, the acceleration of a moving object was provided, and the displacement was required as part of the response. Regular exposure to a range of contexts would contribute to confidence in working with second derivative problems.

Samples

Short response

Paper 1, Questions 13b) and 15b)

The following excerpts are from Question 13b) and Question 15b) from Paper 1. Both questions required algebraic facility to be able to solve them without technology.

Effective student responses:

- determined an appropriate method to expand the squared bracket
- understood that each term in the expanded function needed to be simplified so that the rules of integration could be applied
- were able to work with either decimal values, fractions or both in solving an equation with one unknown within a square root function.

These excerpts have been included:

- to demonstrate the algebraic skills of manipulating powers, expanding brackets and simplifying fractional terms, which were required before being able to perform the required integration
- to demonstrate the effective interchange between fractions and decimals to enable efficient technology-free calculations.

Excerpt 1

$$\int \frac{dy}{dx}\,dx = y$$

$$\int \left(\frac{3x^7-2x}{x^4}\right)^2 dx$$

$$= \int (3x^3 - 2x^{-3})^2\,dx$$

$$= \int (3x^3-2x^{-3})(3x^3-2x^{-3})\,dx$$

$$= \int (9x^6 - 12 + 4x^{-6})\,dx$$

$$= 9\frac{x^7}{7} - 12x + 4\frac{x^{-5}}{-5} + C$$

$$= \frac{9}{7}x^7 - 12x - \frac{4}{5x^5} + C$$

Excerpt 2

$$\frac{1}{5} = 2\sqrt{\frac{0.5 \times 0.5}{n}}$$

$$\frac{1}{10} = \sqrt{\frac{0.5 \times 0.5}{n}}$$

$$\frac{1}{100} = \frac{0.25}{n}$$

$$n = 0.25 \div \frac{1}{100}$$

$$= \frac{1}{4} \times \frac{100}{1}$$

$$= \frac{100}{4} = 25 \text{ people}$$

Paper 1, Question 18

The following excerpt is from Question 18 from Paper 1. It required students to develop an equation based on the cosine rule that included a variable for an unknown height. An approximate value (technology-free item) for the unknown height was to be compared to a given value to make a justified decision regarding the suitability of the storage container.

Effective student responses:

- identified the storage container height as being required and introduced a variable for this
- established expressions for all side lengths of the triangle shown in the diagram, some including the introduced variable
- recognised that the cosine rule would be needed to develop an equation that would connect the triangle side lengths with the one given angle
- solved a trigonometric equation to determine an approximate value for the height of the storage container.

This excerpt has been included:

- as it demonstrates logical organisation within the response where key steps are clear
- to demonstrate an efficient method for estimating a value obtained in the technology-free calculation to enable a comparison to be made.

$AB = \sqrt{4^2 + 3^2}$

$= 5\text{m}$

let h = height of container

$AC = \sqrt{9 + h^2}$

$BC = \sqrt{16 + h^2}$

$AC^2 = 5^2 + BC^2 - 2 \times 5 \times BC \times \cos(60°)$

$9 + h^2 = 25 + 16 + h^2 - 10\sqrt{16 + h^2} \times \frac{1}{2}$

$9 + h^2 = 41 + h^2 - 5\sqrt{16 + h^2}$

$5\sqrt{16 + h^2} = 32$

$\sqrt{16 + h^2} = \frac{32}{5}$

$16 + h^2 = \left(\frac{32}{5}\right)^2$

$h^2 = \left(\frac{32}{5}\right)^2 - 16$

$h^2 = \frac{1024}{25} - \frac{400}{25}$

$h^2 = \frac{624}{25}$

$h = \sqrt{\frac{624}{25}}$

$h \approx 5\text{ m}$

$5\text{m} > 4\text{m}$

$\therefore$ container does meet person's requirements

Paper 1, Question 19

The following excerpt is from Question 19 from Paper 1. It required students to develop a trigonometric model of the seasonal movement of the front edge of a glacier to determine times in the year when the acceleration of the glacier front edge was above a certain value.

Effective student responses:

- used the description of a glacier front edge to determine a model for its position relative to a car park
- understood that the second derivative of the position model was required to determine the acceleration of the glacier front edge
- solved a trigonometric equation to obtain multiple solutions within a calendar year of time.

This excerpt has been included:

- to demonstrate how the information from the context of the question was carefully and accurately used to develop a mathematical model for position
- to demonstrate the use of the second derivative as required in this context
- as it provides evidence of the correct interpretation of the absolute value to solve the trigonometric equation.

Difference in distances $= 280 - 270 = 10$

$\therefore A = \frac{10}{2} = 5$

~~B=~~ Period $= \frac{2\pi}{B}$

$12 = \frac{2\pi}{B}$ let x represent time in months

$B = \frac{\pi}{6}$

no phase shift present $\therefore C = 0$

$D = \frac{280 + 270}{2} = 275$

$\therefore s(x) = 5\sin\left(\frac{\pi}{6}x\right) + 275$

$s'(x) = \frac{5\pi}{6}\cos\left(\frac{\pi}{6}x\right)$

$s''(x) = -\frac{5\pi^2}{36}\sin\left(\frac{\pi}{6}x\right)$

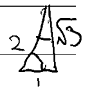
$s''(x)$ = acceleration of front edge

Absolute value of $\frac{5\pi^2\sqrt{3}}{72}$.

as it is a sin graph there will be a negative and positive period

negative period

$-\frac{5\pi^2\sqrt{3}}{72} = -\frac{5\pi^2}{36}\sin\left(\frac{\pi}{6}x\right)$



$\frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{6}x\right)$

$\therefore x = 2, 4, 14, 16, \ldots$

as within domain of year $0 \leq x \leq 12$

$x = 2$ and 4

therefore with negative acceleration they are there for 2 months

positive period

$\frac{5\pi^2\sqrt{3}}{72} = -\frac{5\pi^2}{36}\sin\left(\frac{\pi}{6}x\right)$

$-\frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{6}x\right)$

$x = 8, 10, 20, 22, \ldots$

within domain $x = 8$ and 10

$\therefore$ they will be set up for 2 months when acceration $= \frac{-5\pi^2\sqrt{3}}{72}$ and set up for 2 months when acceleration $= \frac{5\pi^2\sqrt{3}}{72}$. $\therefore$ they will be there for 4 months each calender year. $\therefore$ claim is NOT REASONABLE.

$h^2 = \left(\frac{32}{5}\right)^2 - 16$

$h^2 = \frac{1024}{25} - \frac{400}{25}$

$h^2 = \frac{624}{25}$

$h = \sqrt{\frac{624}{25}}$

$h \approx 5\text{ m}$

$5\text{m} > 4\text{m}$

$\therefore$ container does meet person's requirements

Paper 2, Question 18

The following excerpt is from Question 18 from Paper 2. It required students to use integral calculus techniques to determine displacements and times when a moving object was within the range of a motion sensor.

Effective student responses:

- understood that two integrations were required to convert the given acceleration formula to a displacement formula
- determined the position of the motion sensor along a path
- determined both the displacement and times when the object was being detected
- understood the definition of average velocity.

This excerpt has been included:

- to demonstrate the use of consecutive integrations to establish a displacement formula
- as it provides clear evidence of the logical steps involved in determining positions and associated times while using a graphing calculator
- to demonstrate the use of the average velocity formula and substitutions made to solve the problem.

when $t=0, x=0, v=0$

$a(t) = 3\sin(2t)$

$v(t) = -3 \times \frac{1}{2}\cos(2t) + c$

$= -\frac{3}{2}\cos(2t) + c$

sub $t=0, v=0$

$0 = -\frac{3}{2}\cos(0) + c$

$= -\frac{3}{2} + c$

$c = \frac{3}{2}$

$\therefore v(t) = -\frac{3}{2}\cos(2t) + \frac{3}{2}$

$x(t) = \int v(t)\,dt$

$= -\frac{3}{2} \times \frac{1}{2}\sin(2t) + \frac{3}{2}t + d$

$= -\frac{3}{4}\sin(2t) + \frac{3}{2}t + d$

sub $t=0, x=0$

$0 = -\frac{3}{4}\sin(0) + d$

$d = 0$

$\therefore x(t) = -\frac{3}{4}\sin(2t) + \frac{3}{2}t$

sub $t = 3$ into $x(t)$

using GC: $x =$ ~~1.7096 m~~ 4.7096

using 2m boundaries, will be detected

from $x = 2.7096$ to $x = 6.7096$

avg velocity $= \frac{\text{displacement}}{\text{time}}$

sub $x = 2.7096$ into $x(t)$

$t = 1.689$

sub $x = 6.7096$ into $x(t)$

$t = 4.59$

avg velocity $= \frac{6.7096 - 2.7096}{4.59 - 1.689}$

$= \frac{4}{2.901}$

$= 1.38$ m/s

Additional advice

- Subject matter from Units 1 and 2 is assumed knowledge and may be drawn on, as applicable, in the development of responses to the external examination questions. For instance, in Paper 2, Question 18, average velocity was required. Average rate of change is Unit 2 Topic 4 subject matter. In Paper 1, Question 19, the size of the acceleration was described with reference to absolute value (Unit 1 Topic 5 subject matter). For this reason, revisiting concepts on a regular basis with students would be beneficial.
- Teachers should emphasise the importance of reading examination questions carefully to ensure that students are clear about what is being asked and what information is available in the question. In Paper 1, Question 19, students needed to realise this context required the use of radian measure and not degrees, and that the description of the seasonal glacier movement was important in developing the values of the variables used within the model.
- Teachers are encouraged to explore alternative methods with students, especially when developing responses to problem-solving questions. In Paper 1, Question 18, some students used vector methods (three-dimensional vector lengths and the dot product formula) to approach the problem. Although vector methods were not an expected response, any alternative method that is mathematically correct could be considered an acceptable response to an external assessment question.