General Mathematics SEE marking guide

External assessment 2023

SEE 1: Short response (50 marks)

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 1, 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 1, 2 and/or 3
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 1, 2 and/or 3.

Purpose

This document consists of a marking guide and a sample response.

The marking guide:

- provides a tool for calibrating external assessment markers to ensure reliability of results
- indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- informs schools and students about how marks are matched to qualities in student responses.

The sample response:

- demonstrates the qualities of a high-level response
- has been annotated using the marking guide.

Mark allocation

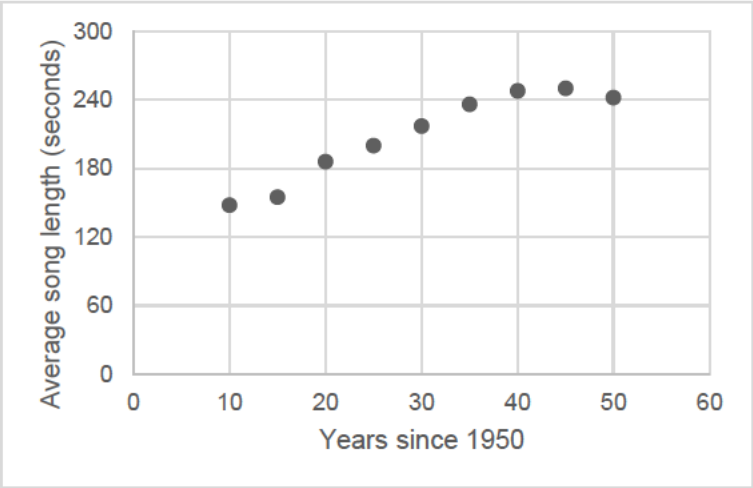
Where a response does not meet any of the descriptors for a question or a criterion, a mark of '0' will be recorded.

Where no response to a question has been made, a mark of 'N' will be recorded.

Allow FT mark/s — refers to 'follow through', where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working — the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not explicitly stated in the student response, but by virtue of subsequent working there is sufficient evidence to award the mark/s.

Marking guide

Q	Sample response	The response:
1		• correctly labels axes and scales for the scatterplot **[1 mark]** • accurately plots the given data points **[1 mark]**

Q	Sample response	The response:
2	Identify variables x = year of release y = song length Define variables Let x = years since 1950 (i.e. x = 15 in 1965) Let y = average song length (seconds) Create model Using a scientific calculator and the data provided in Stimulus 1: Linear equation in the form $y = ax + b$ $a = 2.7367,\ b = 127.01$ $y = 2.7367x + 127.01$	• correctly identifies the explanatory and response variables **[1 mark]** • defines the explanatory and response variables **[1 mark]** • determines the parameters for a linear model using all the data values **[1 mark]** • determines a linear model **[1 mark]**

Q	Sample response	The response:
3	Define variables Let n = years since 1950 (i.e. $n = 15$ in 1965) Let t_n = average song length (seconds) Create geometric model $t_n = t_1 r^{(n-1)}$ Consider 1965: $n = 15$, $t_n = 155$ Equation 1: $155 = t_1 r^{14}$ Consider 1990: $n = 40$, $t_n = 248$ Equation 2: $248 = t_1 r^{39}$ Calculate r: $\frac{\text{Equation 2}}{\text{Equation 1}}$ $\frac{248}{155} = \frac{t_1 r^{39}}{t_1 r^{14}}$ $1.6 = r^{25}$ $r = 1.019$ Calculate t_1: Substitute r into Equation 1 $155 = t_1 \times 1.019^{14}$ $t_1 = 119.13$ Geometric model: $t_n = 119.13 \times 1.019^{(n-1)}$	• correctly defines the explanatory and response variables **[1 mark]** • correctly identifies n and t_n values for one point **[1 mark]** • determines one of the geometric equations **[1 mark]** • identifies n and t_n for a second point **[1 mark]** • determines a second geometric equation **[1 mark]** • provides evidence of solving simultaneous equations **[1 mark]** • determines the value for r **[1 mark]** • provides evidence of substituting r into one of the equations **[1 mark]** • determines the value for t_1 **[1 mark]** • determines a geometric model **[1 mark]** • shows logical organisation, communicating key steps **[1 mark]**

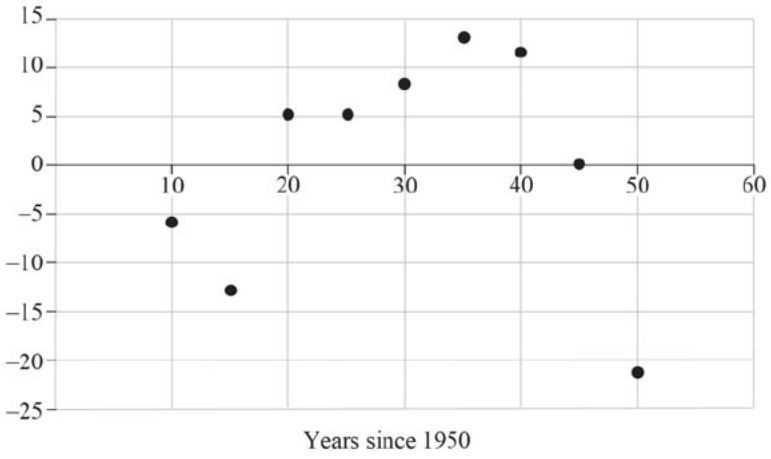
Q	Sample response	The response:
4a)	Residuals for linear model	• correctly provides an appropriately organised table **[1 mark]**

x	Time (A)	$y = 2.7367x + 127.01$ (M)	Residual ($A - M$)
10	148	154.377	–6.377
15	155	168.0605	–13.0605
20	186	181.744	4.256
25	200	195.4275	4.5725
30	217	209.111	7.889
35	236	222.7945	13.2055
40	248	236.478	11.522
45	250	250.1615	–0.1615
50	242	263.845	–21.845

- calculates the predicted values for five values of x **[1 mark]**
- calculates the predicted values for the remaining values of x **[1 mark]**

- calculates the residuals for all data values with the model **[1 mark]**

Residual plot for linear model



- correctly labels the axes and chooses appropriate scales on at least one residual plot **[1 mark]**

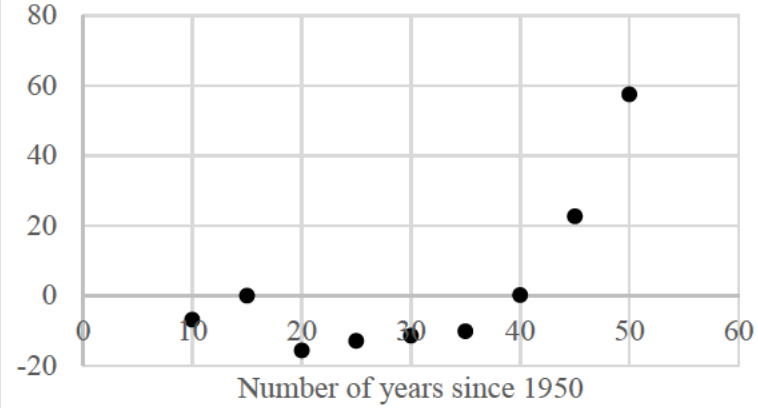
- plots the points on the residual plot for the model **[1 mark]**

Residuals for geometric model

Q	Sample response	The response:

x	Time (*A*)	$t_n = 119.13 \times 1.019^{(n-1)}$ (*M*)	Residual (*A* – *M*)
10	148	141.12	6.88
15	155	155.05	–0.05
20	186	170.35	15.65
25	200	187.16	12.85
30	217	205.62	11.38
35	236	225.91	10.09
40	248	248.21	–0.21
45	250	272.70	–22.70
50	242	299.61	–57.61

Residual plot for geometric model



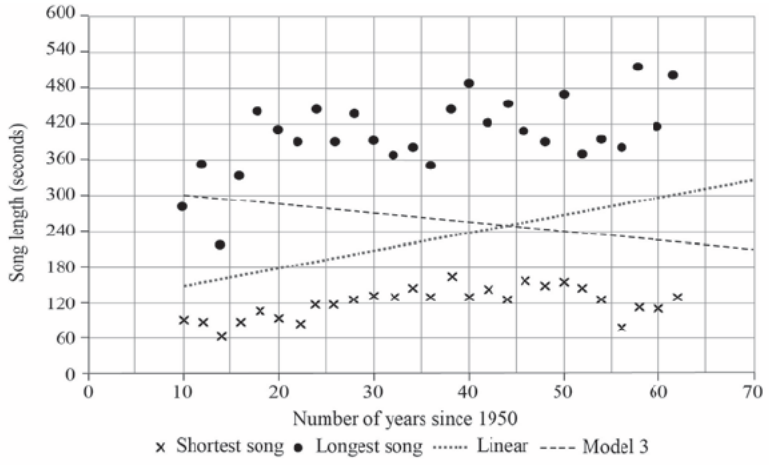
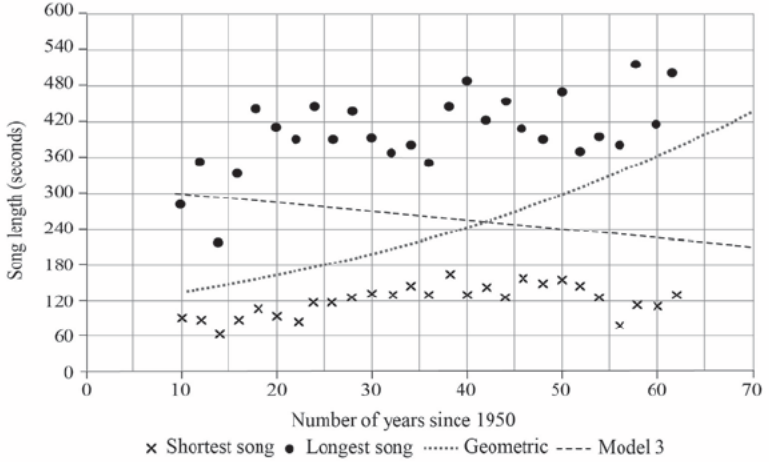
The magnitude of the residuals tends to be smaller for the residual model. The residual plots for both models look random and results are on both sides of the x-axis.

- correctly provides an appropriately organised table **[1 mark]**
- calculates the predicted values for five values of x **[1 mark]**
- calculates the predicted values for the remaining values of x **[1 mark]**
- calculates the residuals for all data values with the model **[1 mark]**
- accurately plots the points on the residual plot for the model **[1 mark]**
- considers the results of the residual plot with respect to randomness of residuals **[1 mark]**
- considers the results of the residual plot with respect to size of residuals **[1 mark]**
- shows logical organisation, communicating key steps **[1 mark]**

4b)	The linear model is more valid	• states the more valid model **[1 mark]**

Q	Sample response	The response:
5	Linear model from Question 4b): $y = 2.74x + 127$ [1] Model 3: $y = -1.46x + 315.2$ [2] Using substitution, equating [1] and [2]: $2.74x + 127 = -1.46x + 315.2$ $4.2x = 188.2$ $x = 44.8$ Substituting x in [1]: $y = 2.74 \times 44.8 + 127$ $= 249.8$ In **1994**, the average song length will be the same (249.8 s) for both models.	• equates the two equations **[1 mark]** • groups like terms **[1 mark]** • solves for x **[1 mark]** • solves for y **[1 mark]** • identifies the year when the average song lengths are equal **[1 mark]**

<table>
<tr><th>Q</th><th>Sample response</th><th>The response:</th></tr>
<tr><td>6a)</td><td>Linear model (if selected as most valid model)
<table>
<tr><th>x</th><th>Time (A)</th><th>$y = 2.7367x + 127.01$ (M)</th><th>Residual (A – M)</th></tr>
<tr><td>50</td><td>242</td><td>263.85</td><td>–21.85</td></tr>
<tr><td>55</td><td>232</td><td>277.53</td><td>–45.53</td></tr>
<tr><td>60</td><td>231</td><td>291.21</td><td>–60.21</td></tr>
<tr><td>65</td><td>223</td><td>304.90</td><td>–81.90</td></tr>
<tr><td>70</td><td>210</td><td>318.58</td><td>–108.58</td></tr>
</table>
Geometric model (if selected as most valid model)
<table>
<tr><th>x</th><th>Time (A)</th><th>$t_n = 119.13 \times 1.019^{(n-1)}$ (M)</th><th>Residual (A – M)</th></tr>
<tr><td>50</td><td>242</td><td>299.61</td><td>–57.61</td></tr>
<tr><td>55</td><td>232</td><td>329.18</td><td>–97.18</td></tr>
<tr><td>60</td><td>231</td><td>361.66</td><td>–130.66</td></tr>
<tr><td>65</td><td>223</td><td>397.35</td><td>–174.35</td></tr>
<tr><td>70</td><td>210</td><td>436.56</td><td>–226.56</td></tr>
</table></td><td>• calculates the predicted values for each year for five years [1 mark]
• calculates the residuals for all data values with the model [1 mark]</td></tr>
<tr><td>6b)</td><td>The most valid model for the period 2000–2020 is model 3.
This is because the residuals for model 3 are small and random, while the residuals for the model from Question 4b) are large and becoming larger with time.</td><td>• identifies model 3 as best model for time period [1 mark]
• provides valid justification for selection of model [1 mark]</td></tr>
</table>

Q	Sample response	The response:
7a)	The linear model  The geometric model  	• correctly sketches model 3 **[1 mark]** • sketches model selected in Question 4b) **[1 mark]** • adds key **[1 mark]**

Q	Sample response	The response:
7b)	For the years 1960–2000, the best model is the linear model $y = 2.7367x + 127.01$. For the years 2000–2020, the best model is model 3 $y = -1.46x + 315.2$. Until 2000, music was recorded on physical devices, so technology increased the amount of music that could be stored. After 2000, streaming services were introduced that potentially disrupted song length. The model described will fit between the longest and shortest songs recorded.	• identifies a reasonable model for the years 1960–2020 **[1 mark]** • uses Stimulus 4 to justify selection **[1 mark]** • uses Question 7a) to justify selection **[1 mark]**
7c)	Using model 3, the average song length in 2050 will be 169.2 seconds. Artists will potentially receive more money for shorter songs, so it makes sense that the average song will be fairly short. The predicted song is nearly three minutes long. It would still be possible to incorporate the features that make a song popular into a song this long, so the answer is reasonable.	• calculates average song length using the model chosen in Question 7b) **[1 mark]** • uses Stimulus 4 to justify reasonableness of prediction **[1 mark]** • uses Stimulus 5 to justify reasonableness of prediction **[1 mark]**